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IDA PAPER P-1373

ANALYTICAL REPRESENTATIONS OF BLAST DAMAGE FOR SEVERAL TYPES OF TARGETS

Leo A. Schmidt, Jr.

October 1978

Prepared for

Defense Civil Preparedness Agency

INSTITUTE FOR DEFENSE ANALYSES
PROGRAM ANALYSIS DIVISION

IDA Log No. HQ 78-20712

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) ANALYTICAL REPRESENTATIONS OF BLAST DAMAGE FOR SEVERAL TYPES OF TARGETS	5. TYPE OF REPORT & PERIOD COVERED Interim Report
	6. PERFORMING ORG. REPORT NUMBER IDA Paper P-1373
7. AUTHOR(s) Leo A. Schmidt, Jr.	8. CONTRACT OR GRANT NUMBER(s) DCPA01-77-C-0215
9. PERFORMING ORGANIZATION NAME AND ADDRESS INSTITUTE FOR DEFENSE ANALYSES 400 Army-Navy Drive Arlington, Virginia 22202	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit 4114G
11. CONTROLLING OFFICE NAME AND ADDRESS Defense Civil Preparedness Agency Washington, D.C. 20301	12. REPORT DATE October 1978
	13. NUMBER OF PAGES 72
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report) UNCLASSIFIED
	15a. DECLASSIFICATION DOWNGRADING SCHEDULE N/A

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

blast damage, target complexes, nuclear attack, damage laws, damage functions

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper is concerned with the "Square Root Damage Law"--an analytical method of computing blast damage on target complexes. The Law assumes that target value is a continuous function of position, and replaces a set of individual weapons attacking a target by a weapon density function that is optimized to give maximum damage. This basic approach is applied to several types of target damage functions and methods of weapon targeting.

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This paper is concerned with the "Square Root Damage Law"--an analytical method of computing blast damage on target complexes. The Law assumes that target value is a continuous function of position, and replaces a set of individual weapons attacking a target by a weapon density function that is optimized to give maximum damage. This basic approach is applied to several types of target damage functions and methods of weapon targeting.

Attention is restricted to cases where the results can be presented in relatively simple analytical form. The derivations are presented in some detail to illustrate the nature of the results obtained. Charts of damage are presented for sets of dimensionless parameters that govern the damage sensitivity to weapon usage. As a practical application, they can form analytical bases for explaining the results obtained from detailed numerical calculations of the damage for a particular target from a specific set of weapon aimpoints.

Chapter I

INTRODUCTION

In assessing the effects of nuclear attack upon target complexes, attention often is restricted to blast effects, or more precisely to effects where the probability of damage is taken only as a function of the distance from a weapon, independent of direction. In many cases the target complex is assumed to consist of a set of discrete target elements; each element has an overall probability of damage obtained by combining the probabilities of damage from separate individually located weapons; and overall damage is calculated by summing the effects on each target element. While this type of calculation is the natural way of assessing blast damage from a particular attack, the results are strictly numerical and no analytical insight is gained from such a calculation.

Analagous to the dichotomy between particulate and continuum mechanics, an approach to casualty estimation can be developed where target elements are replaced by a target value as a continuous function of position, and weapon locations are replaced by a weapon density function. When the target value function is a circular normal function, the local fraction of damage is an exponential function of weapon density, and weapons are optimally targeted against value, a simple expression can be obtained for overall damage to the target complex as a function of weapon usage. This expression has become known as the "Square Root Damage Law." A derivation of this law, using a somewhat more general expression for probability of damage as a function of weapon usage, is given by Galiano and Everett.[1] A comparison of the results of this formula with the results of calculations against individual target elements is given by Schmidt[2] for a variety of cases. A surprising degree of agreement of the two methods is evident.

Despite the venerability of the Square Root Damage Law, no efforts have been made to apply this general approach to other situations. That is precisely the intent of the present paper. In this attempt, the algebraic structure is presented in some detail to assist in understanding the nature of the results.

Chapter II presents a derivation of the damage laws when the population is in a circular normal distribution, or uniformly distributed in a disk or in a ring. Results are obtained from situations where the targeting is optimal against one type of target value distribution or hardness, but the actual hardness is of a different type. Such results are of interest when two separate types of target values are collocated, e.g., where the targeting is optimized against the industry in a target complex and the evaluation is against the population.

Following this, a more generalized derivation of the damage law allows damage calculations for value distributions that are a function only of radial distance, but are arbitrary in shape. Several applications of this derivation are presented. Finally, some factors influencing the shape of the damage functions are presented.

[1] Robert F. Galiano and Hugh Everett, III, *Defense Models IV, Family of Damage Functions for Multiple Weapon Attacks*, Lamda Corp., Paper 6, March 1967.

[2] Leo A. Schmidt, *A Sensitivity Analysis of Urban Blast Fatality Calculations*, Institute for Defense Analyses, IDA P-762, January 1971.

Chapter II

ANALYSES

A. SQUARE ROOT DAMAGE LAW DERIVATION

This section will rederive the Square Root Damage Law in some detail as a basic algebraic theme from which variations will be taken. In so doing, the algebraic structure rather than the logical basis is of prime interest.

Assume a population, V_0, is a circular normal distribution so that the population density at any distance r from the center of the distribution is

$$V(r) = V_0/2\pi\alpha^2 \exp\left(-r^2/2\alpha^2\right) . \tag{1}$$

In subsequent discussions, the distribution of the targeted value, V_T, will usually be different than the actual one; in describing the former, α will be replaced by β; in the latter, α will be replaced by σ.

Assume a weapon density ω causes a fraction of fatalities $F(\omega)$ and that the functional form is[1]

$$F(\omega) = 1 - e^{-k\omega} . \tag{2}$$

The damage at any point is

$$h(r,\omega) = V(r) \cdot F(\omega) . \tag{3}$$

Through the introduction of a Lagrange Multiplier λ, one can

[1] For a discussion of the relation of a weapon density to a discrete set of targeted weapons, and how to determine values of the constant k, see Schmidt, *op. cit.*

see that for an allocation that maximizes total return

$$H = \int h(r,\omega)dA \tag{4}$$

as a function of total weapon usage

$$W = \int \omega(r)dA \tag{5}$$

we have

$$\partial h(r,\omega)/\partial\omega = \lambda ,$$

whenever $\omega > 0$. Since

$$\partial h(r,\omega)/\partial\omega = Vke^{-k\omega} ,$$

we have

$$e^{-k\omega} = \lambda/kV , \tag{6}$$

or

$$\omega = \begin{cases} 0 , & kV/\lambda < 1 \\ 1/k \ln(kV/\lambda), & kV/\lambda \geq 1 , \end{cases} \tag{7}$$

and

$$F = 1 - \lambda/kV \tag{8}$$

when $\omega \neq 0$. At this point it is convenient to define

$$\gamma = 2\pi\alpha^2\lambda/kV_0 . \tag{9}$$

γ is proportional to the Lagrange Multiplier λ. Now we can write

$$\omega = \begin{cases} 0 , & \exp(-r^2/2\alpha^2) < \gamma \\ 1/k \ln(1/\gamma)\exp(-r^2/2\alpha^2) , & \exp(-r^2/2\alpha^2) \geq \gamma , \end{cases} \tag{10}$$

and

$$F = 1 - \gamma \exp(r^2/2\alpha^2) . \tag{11}$$

The weapon usage W is now found from

$$W = \int_0^{r_c} 1/k \ln(1/\gamma \exp(-r^2/2\alpha^2))\, 2\pi r\, dr ,$$

where r_c solves

$$\exp(-r_c^2/2\alpha^2) = \gamma . \qquad (12)$$

This is readily found to be

$$W = 2\pi/k\, [\ln(1/\gamma) \cdot r_c^2/2 - r_c^4/8\alpha^2] ,$$

$$W = 2\pi/k\, [\ln(1/\gamma) \cdot r_c^2/4] ,$$

$$W = (\pi/k)\alpha^2 \ln^2(1/\gamma) , \qquad (13)$$

using the definition of r_c twice. A normalized weapon usage is introduced by

$$X = kW/\pi\alpha^2 , \qquad (14)$$

from which

$$X = \ln^2(1/\gamma)$$

and

$$\gamma = \exp(- \sqrt{X}\ ) . \qquad (15)$$

A critical point in achieving a final expression for survivors directly in X, rather than two expressions in the parameter γ, is solving for γ in terms of X.

The total fatalities are found from

$$H = \int_0^{r_c} (1 - \gamma \exp(r^2/2\alpha^2))\, V_0/2\pi\alpha^2 \exp(-r^2/2\alpha^2)\, 2\pi r\, dr .$$

This is readily integrated to

$$H = V_0 \left(1 - \exp(-r_c^2/2\alpha^2)\right) - \gamma V_0 \, r_c^2/2\alpha^2 \ .$$

The fatalities are expressed as a fraction and the definition of r_c is used to give

$$H/V_0 = 1 - \gamma - \gamma \ln(1/\gamma)$$

$$H/V_0 = 1 - \gamma(1 + \ln(1/\gamma)) \ . \qquad (16)$$

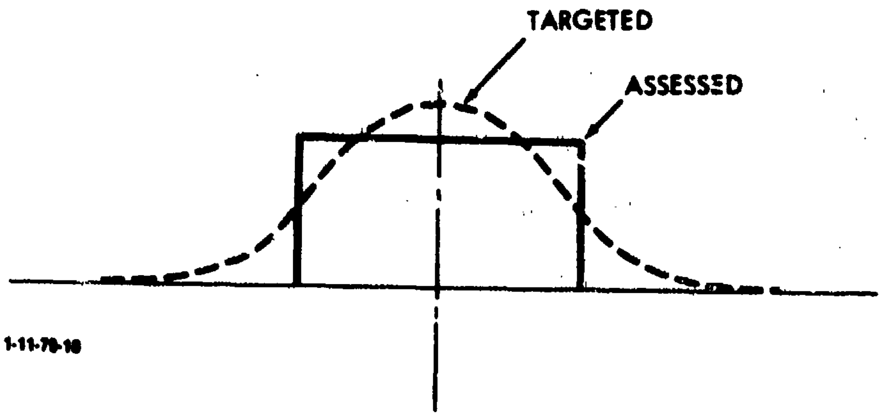
Now using the expression for γ found from integrating the weapon usage,

$$H/V_0 = 1 - \exp(-\sqrt{X}\,)(1 + \sqrt{X}\,) \ . \qquad (17)$$

If S is the fraction of survivors, then

$$S = \exp(-\sqrt{X}\,)(1 + \sqrt{X}\,) \ . \qquad (18)$$

B. SURVIVORS FROM POPULATION IN A UNIFORM DISK TARGETED AS A GAUSSIAN POPULATION



Assume a population V_0, for which damage is to be assessed, is uniformly distributed in a disk of radius R with population density

$$V(r) = \begin{cases} V_0/\pi R^2, & r \le R , \\ 0, & r > R . \end{cases} \qquad (19)$$

The targeting is assumed to be against a Gaussian distributed population given by

$$V = V_T/2\pi\beta^2 \exp(-r^2/2\beta^2) \qquad (20)$$

with a damage function as defined in the square root law derivation.

To relate the assessed population to the targeting, call

$$\psi = R^2/2\beta^2 . \qquad (21)$$

Since the targeting is identical to the previous section, then

$$\chi = \ln^2(1/\gamma),$$

using the derivation of the previous section with α replaced by β.

To compute damage, two cases must be considered, namely

Case I: $r_c \le R$

Case II: $r_c > R$

where r_c, as before, is the limiting radius for $\omega > 0$. For Case I, using Equation (11)

$$H = \int_0^{r_c} V_0/\pi R^2 \cdot (1 - \gamma \exp(r^2/2\beta^2))\ 2\pi r\, dr ,$$

thus

$$H/V_0 = 2/R^2 \left[r_c^2/2 - \beta^2\gamma(\exp(r_c^2/2\beta^2) - 1)\right] .$$

Now using the definition of r_c,

$$H/V_0 = 2\beta^2/R^2\ [\ln(1/\gamma) - \gamma/\gamma + \gamma] = 1/\varphi\ [\ln(1/\gamma) + \gamma - 1] .$$

In terms of X, using Equation (15)

$$H/V_0 = 1/\varphi\ [\exp(-\sqrt{X}) + \sqrt{X} - 1] . \tag{22}$$

For Case II

$$H = \int_0^R V_0/\pi R^2 \cdot (1 - \gamma \exp(r^2/2\beta^2))\ 2\pi r\, dr .$$

Thus

$$H/V_0 = 2/R^2\ [R^2/2 - \beta^2\ \gamma \exp(R^2/2\beta^2) + \beta^2\gamma] .$$

So

$$H/V_0 = 1/\varphi\ [\varphi - \gamma \exp(\varphi) + \gamma] , \tag{23}$$

or

$$H/V_0 = 1/\varphi\ [\varphi + \exp(-\sqrt{X})(1 - \exp(\varphi))] . \tag{24}$$

To distinguish between Case I and Case II, we notice that $r_c = R$ at the border between the two cases; then, by definition of r_c

$$\exp(-R^2/2\beta^2) = \gamma \tag{25}$$

from which, using Equation (15), at a critical X

$$X_c = \varphi^2 .$$

Summarizing, the fraction fatalities are given by

$$H/V_0 = \begin{cases} 1/\varphi\ [\exp(-\sqrt{X}) + \sqrt{X} - 1], & X \le \varphi^2 \\ 1/\varphi\ [\varphi + \exp(-\sqrt{X})(1 - \exp(\varphi))], & X > \varphi^2 \end{cases} . \tag{26}$$

In terms of survivors,

$$S = \begin{cases} 1 + 1/\varphi - 1/\varphi(\exp(-\sqrt{X}) + \sqrt{X}), & X \le \varphi^2 \\ \exp(-\sqrt{X})(\exp(\varphi) - 1)/\varphi, & X > \varphi^2 \end{cases} \quad . \qquad (27)$$

The fraction of survivors as a function of X is presented in Figure 1 for various values of φ. The dashed line separates the two regions where the attack radius is less than or greater than the target radius of the assessed population. Large values of φ have more survivors at constant X, in part simply because the attack is the same and the target is larger.

To express survivors in terms of weapon usage normalized to the assessed rather than the targeted population, let σ be the standard deviation of the targeted population about the targeted population disk's center; i.e.,

$$\sigma^2 = \int_0^R r^2 \cdot \frac{2\pi r}{\pi R^2} dr \ ,$$

so

$$\sigma = R/\sqrt{2} \ . \qquad (28)$$

If φ is expressed in terms of σ, we have $\varphi = \sigma^2/\beta^2$. To normalize in terms of the actual target, let

$$Y = kW/\pi\sigma^2 \ , \qquad (29)$$

then

$$Y = X/\varphi \ .$$

Survivors can be expressed by replacing X in the previous formula by φY, i.e.,

$$S = \begin{cases} 1 + 1/\varphi - 1/\varphi\ (\exp(-\sqrt{\varphi Y} + \sqrt{\varphi Y}\ ), & Y \le \varphi \\ \exp(-\sqrt{\varphi Y}\ )(\exp(\varphi - 1))/\varphi, & Y > \varphi . \end{cases} \qquad (30)$$

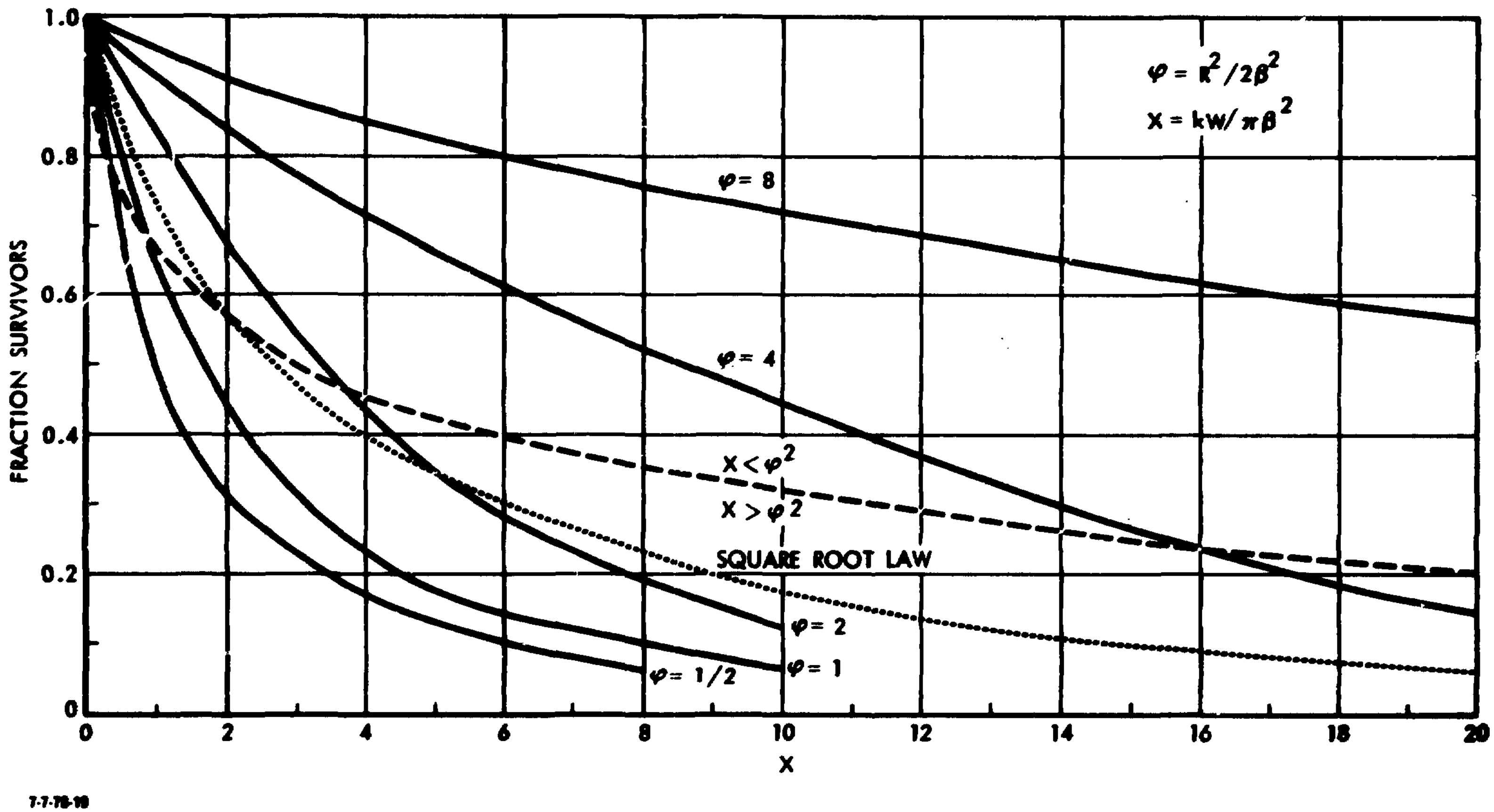


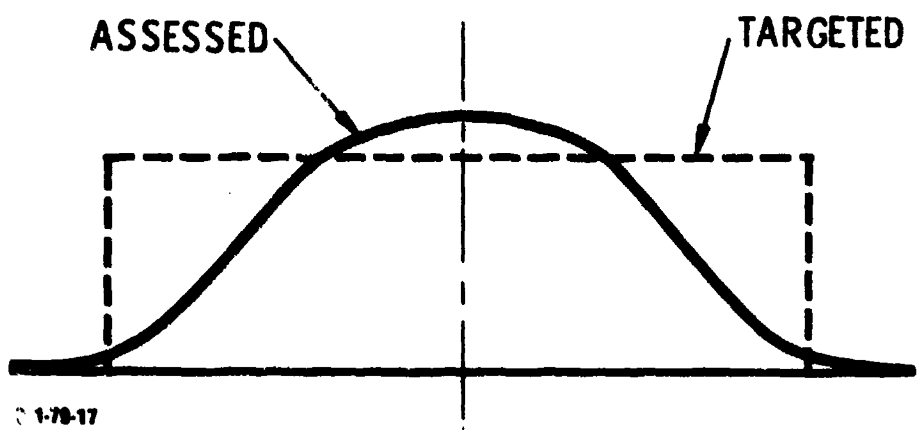
Figure 1. FRACTION SURVIVORS FOR POPULATION IN DISK OF RADIUS R WITH GAUSSIAN TARGETING FOR VARIOUS VALUES OF φ WITH WEAPONS NORMALIZED BY TARGETING-- $\varphi = R^2/2\beta^2$, $X = kW/\pi\beta^2$

Survivors as a function of Y are presented in Figure 2. Since the normalization is by assessed population, the lower curves occur with more efficient targeting. It is interesting to notice that for $\psi = 8$, where the assessed population is much more dispersed than target population, the targeting is more efficient for heavy attacks ($Y = 10$) than when $\psi = 1$ with the two populations matched. This occurs for heavy attacks, since many weapons are expended outside the assessed population disk where $\psi = 1$. An optimal attack against a disk has a uniform weapon density. The survivors are readily seen to be (see(35)):

$$S = e^{-Y/2} \quad . \tag{31}$$

The curve labeled 'optimal targeting' in Figure 2 exhibits this formula and is a lower bound for any of the other curves.

C. SURVIVORS FROM A GAUSSIAN POPULATION TARGETED AS A UNIFORM DISK



Assume a population, V_0, is Gaussian distributed with standard deviation σ

$$V(r) = V_0/2\pi\sigma^2 \exp(-r^2/2\sigma^2) \quad . \tag{32}$$

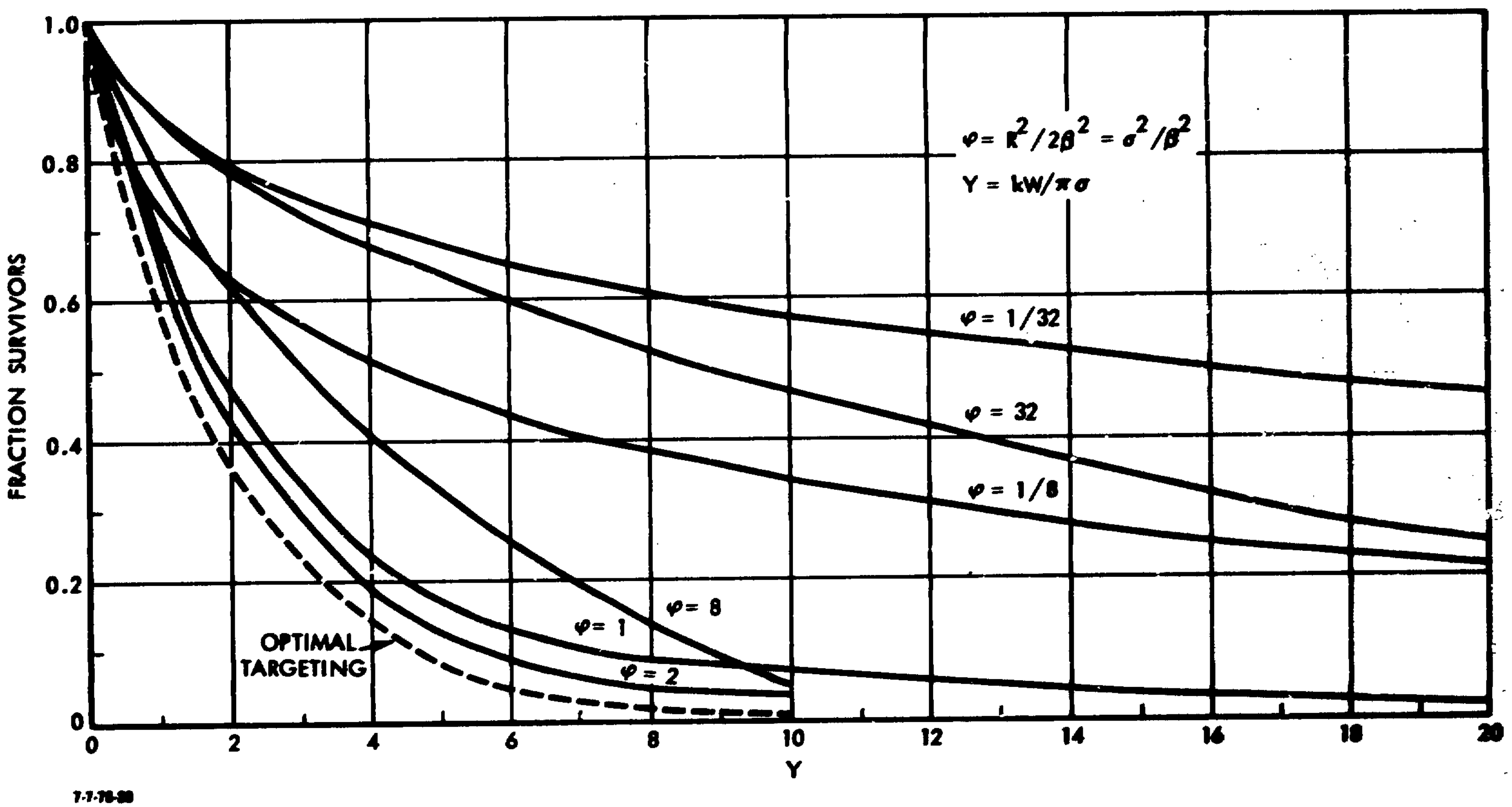


Figure 2. FRACTION SURVIVORS FOR POPULATION IN DISK FOR GAUSSIAN TARGETING FOR VARIOUS VALUES OF φ WITH WEAPONS NORMALIZED BY POPULATION--$\varphi = R^2/2\beta^2 = \sigma^2/\beta^2$, $Y = kW/\pi\sigma^2$

The targeting is assumed to be against a disk of radius R with

$$V(r) = \begin{cases} V_T/\pi R^2 & r \le R , \\ 0 & r > R . \end{cases} \qquad (33)$$

Let β be the standard deviation of the disk about the center as in the last section, so

$$\beta = R/\sqrt{2} .$$

As before, define $\varphi = \sigma^2/\beta^2$ to describe the relative size of the two populations. It is clear that the optimum targeting against a uniform value, is a constant weapon density, thus

$$W = \pi R^2 \omega ,$$

then

$$F(\omega) = 1 - \exp(-kW/\pi R^2) .$$

Define X by

$$X = kW/\pi\beta^2 , \qquad (34)$$

then

$$F = 1 - \exp(-X/2) . \qquad (35)$$

The damage H is given by

$$H = \int_0^R (1 - \exp(-X/2))\, V_0/2\pi\sigma^2 \exp(-r^2/2\sigma^2)\, 2\pi r\, dr .$$

This is readily seen to give

$$H/V_0 = (1 - \exp(-X/2))\, (1 - \exp(-R^2/2\sigma^2)) ,$$

or

$$H/V_0 = [1 - \exp(-X/2)]\, [1 - \exp(-1/\varphi)] . \qquad (36)$$

For survivors

$$S = \exp(-X/2)\ [1 - \exp(-1/\varphi)] + \exp(-1/\varphi)\ . \qquad (37)$$

To express survivors in terms of the assessed population, we define

$$Y = kW/\pi\sigma^2\ . \qquad (38)$$

Then

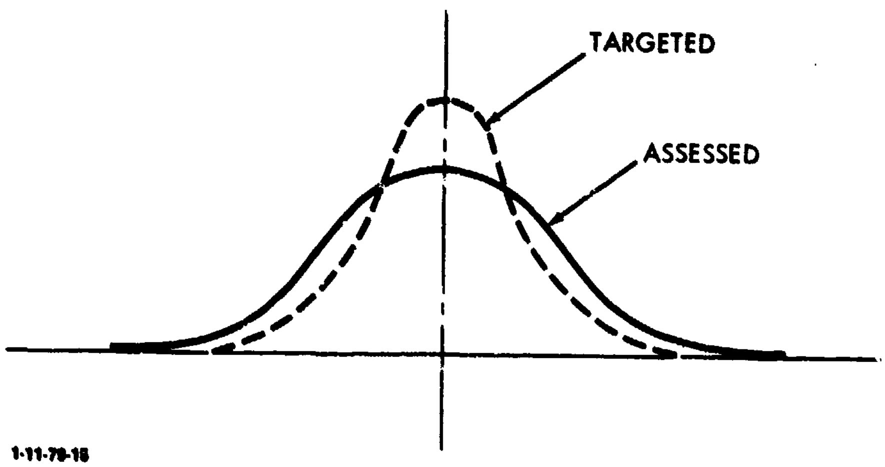
$$Y = X/\varphi \qquad (39)$$

so

$$S = \exp(-Y\varphi/2)\ [1 - \exp(-1/\varphi)] + \exp(-1/\varphi)\ . \qquad (40)$$

The fraction of survivors in Figure 3 is for weapons normalized by targeting and in Figure 4 for weapons normalized by population; also Figure 4 shows the Square Root Damage Law which represents the optimal attack. As such, it must be a lower bound for other attacks. As can be seen for $\varphi = 1/2$, the actual survivors are close to this lower bound for almost all Y.

D. SURVIVORS FROM A GAUSSIAN POPULATION TARGETED AS A GAUSSIAN POPULATION OF A DIFFERENT DISPERSION



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Assume a population, V_0, is Gaussian distributed with standard deviation σ so

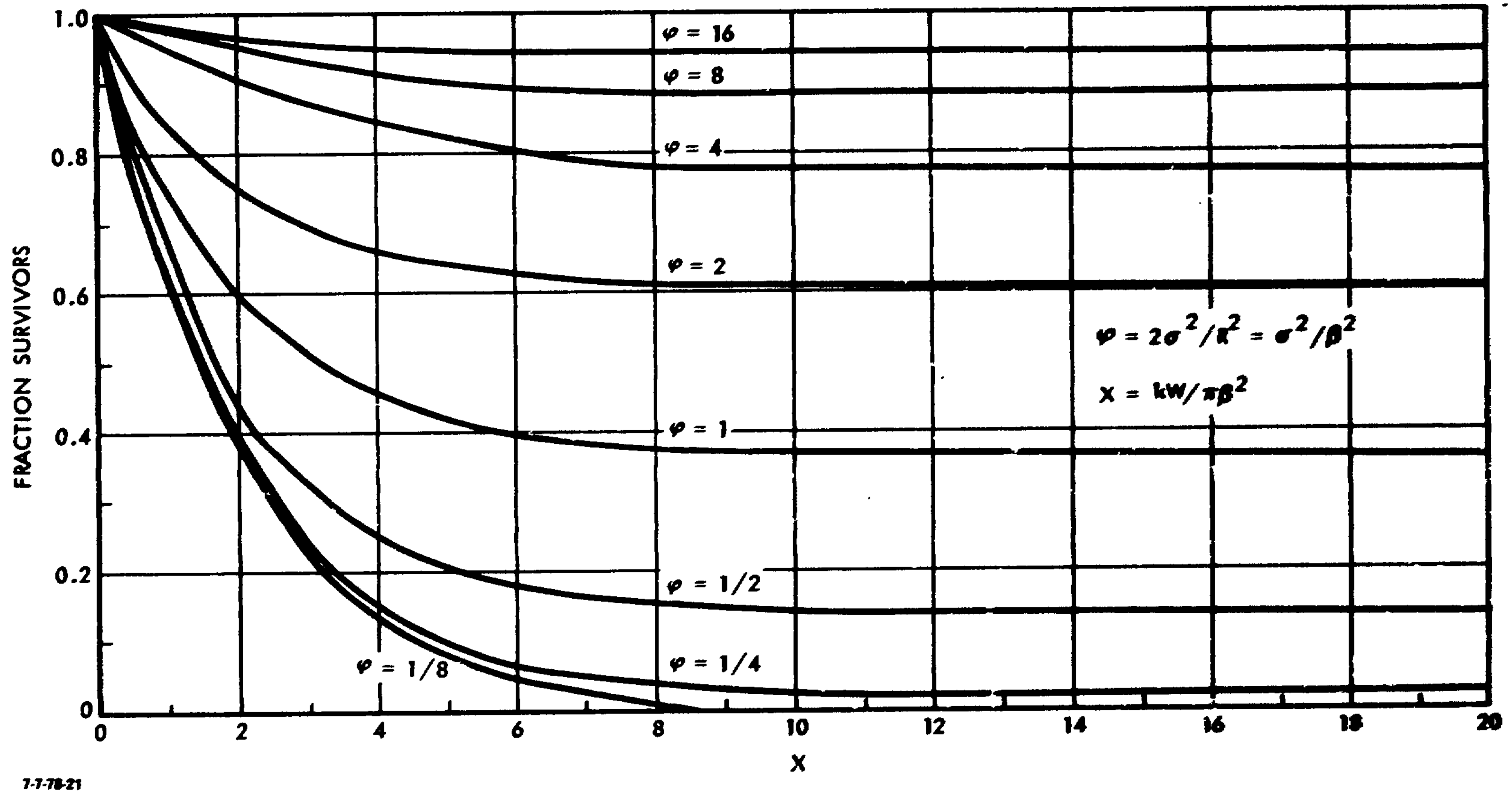

Figure 3. FRACTION SURVIVORS FOR GAUSSIAN POPULATION FOR TARGETING OF UNIFORM DISKS OF RADIUS R WITH WEAPONS NORMALIZED BY TARGET $\varphi = 2\sigma^2/R^2 = \sigma^2/\beta^2$, $X = kW/\pi\beta^2$

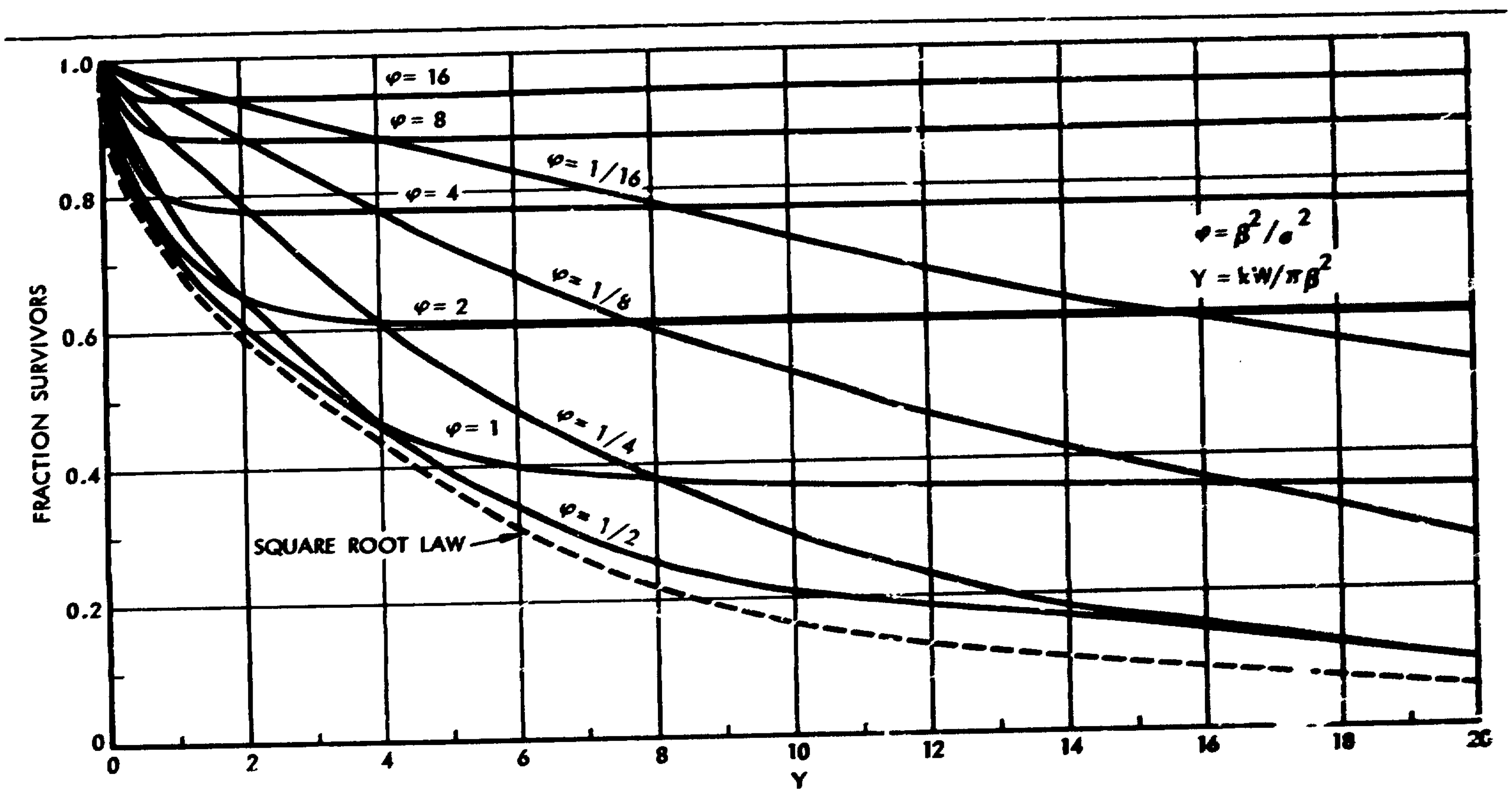

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Figure 4. FRACTION SURVIVORS FOR GAUSSIAN POPULATION FOR TARGETING OF UNIFORM DISK WITH WEAPONS NORMALIZED BY POPULATION $\varphi = \beta^2/\sigma^2$, $Y = kW/\pi\beta^2$

$$V(r) = V_0/2\pi\sigma^2 \exp(-r^2/2\sigma^2) \ . \tag{41}$$

The targeting is assumed to be against a Gaussian distributed population given by

$$V(r) = V_T/2\pi\beta^2 \exp(-r^2/2\beta^2) \ . \tag{42}$$

As before, let the ratio of the two dispersions be specified by $\varphi = \sigma^2/\beta^2$. The targeted population requires a weapon usage as computed for the Square Root Damage Law, so, by Equation (15),

$$X = \ln^2(1/\gamma) \ .$$

Moreover, for F we again have

$$F = \begin{cases} 1 - \gamma \exp(r^2/2\beta^2) \ , & r \le r_c \\ 0 \ , & r > r_c \end{cases} \tag{43}$$

where r_c solves

$$\exp(-r_c^2/2\beta^2) = \gamma \ . \tag{44}$$

The damage H is given by

$$H = \int_0^{r_c} [1 - \gamma \exp(r^2/2\beta^2)] \ V_0/2\pi\sigma^2 \exp(-r^2/2\sigma^2) \ 2\pi r \ dr \ .$$

Integrating gives

$$H/V_0 = [1 - \exp(-r_c^2/2\sigma^2)]$$
$$- \gamma/(1 - \varphi)[1 - \exp(r_c^2/2\beta^2) \cdot \exp(-r_c^2/2\sigma^2)] \ .$$

Now using

$$\exp(-r_c^2/2\sigma^2) = (\exp(-r_c^2/2\beta^2))^{1/\varphi}$$

and recalling the definition of r_c gives

$$H/V_0 = (1 - \gamma^{1/\varphi}) - \gamma/(1 - \varphi)[1 - \gamma^{1/\varphi} \cdot 1/\gamma] . \tag{45}$$

Now defining X as $kW/\pi\beta^2$, and substituting for γ using Equation (15),

$$H/V_0 = 1 - \exp(- \sqrt{X}/\varphi)$$
$$-[\exp(- \sqrt{X})/(1 - \varphi)][1 - \exp(- \sqrt{X}/\varphi)/\exp(- \sqrt{X})] . \tag{46}$$

For survivors,

$$S = (\varphi/\varphi-1) \exp(- \sqrt{X}/\varphi) - \exp(- \sqrt{X}/(\varphi-1)) . \tag{47}$$

To normalize by the actual population, let

$$Y = kW/\pi\sigma^2 . \tag{48}$$

So

$$Y = X/\varphi . \tag{49}$$

Then

$$S = (\varphi/\varphi-1) \exp(-\sqrt{Y/\varphi}) - 1/(\varphi-1) \exp(- \sqrt{Y\varphi}) . \tag{50}$$

It is interesting to notice that if φ is replaced by $1/\varphi$, the same formula is obtained. This implies that if the target population is either too small by a factor of X or too large by a factor of X, relative to the actual population, the same number of survivors is obtained. The fraction of survivors is presented in Figure 5 with weapons normalized for targeting and in Figure 6 with weapons normalized for assessed population.

E. SQUARE ROOT INJURY LAW

Assume a population is Gaussian distributed so that

$$V(r) = V_0/2\pi\sigma^2 \exp(-r^2/2\sigma^2) . \tag{51}$$

Assume the population is targeted so that

$$F(\omega) = 1 - e^{-k\omega} . \tag{52}$$

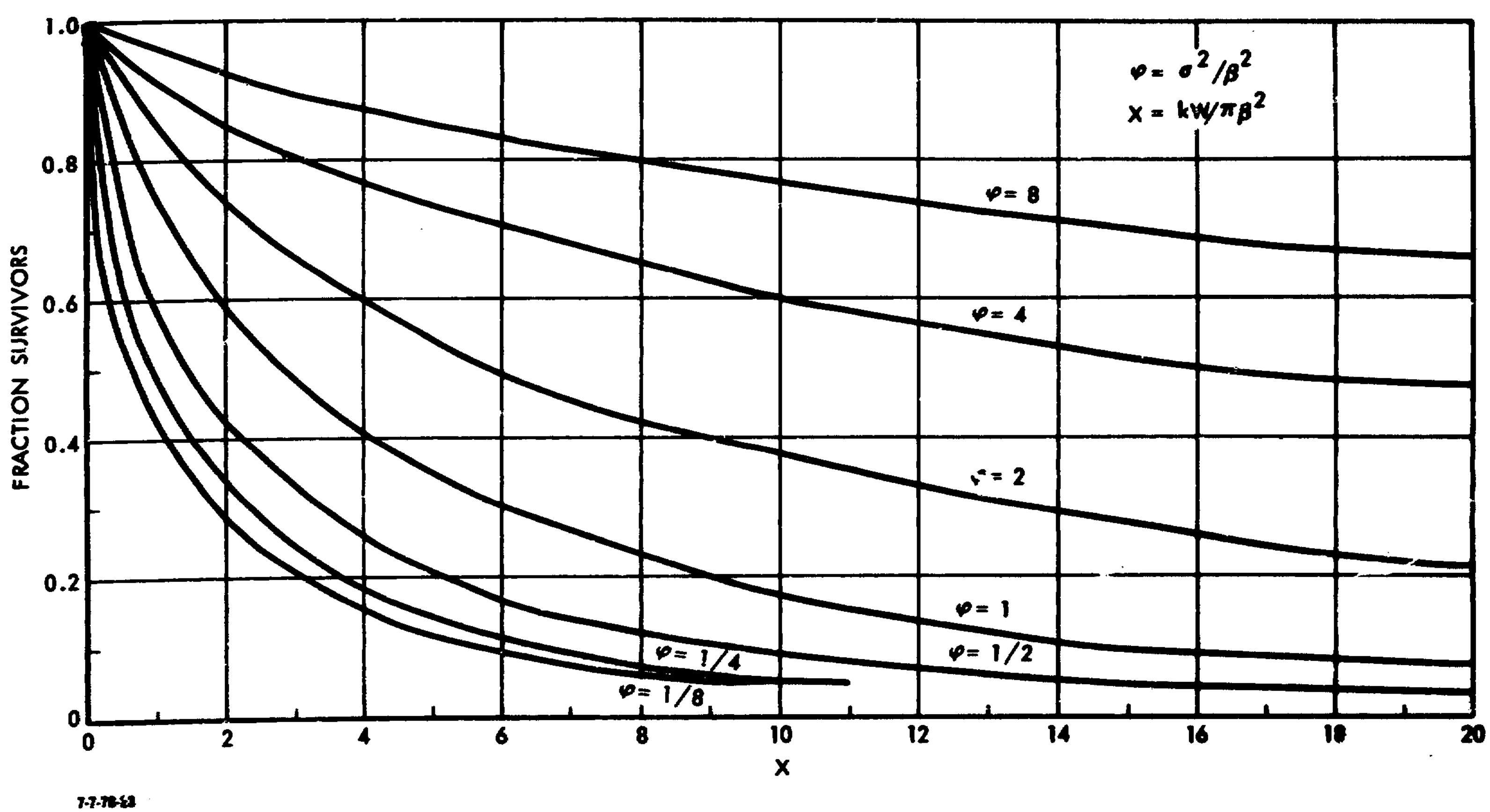

Figure 5. FRACTION SURVIVORS OF GAUSSIAN POPULATION WITH STANDARD DEVIATION β WITH TARGETING AGAINST GAUSSIAN POPULATION WITH STANDARD DEVIATION σ WITH WEAPONS NORMALIZED FOR POPULATION $\varphi = \sigma^2/\beta^2$, $X = kW/\pi\beta^2$

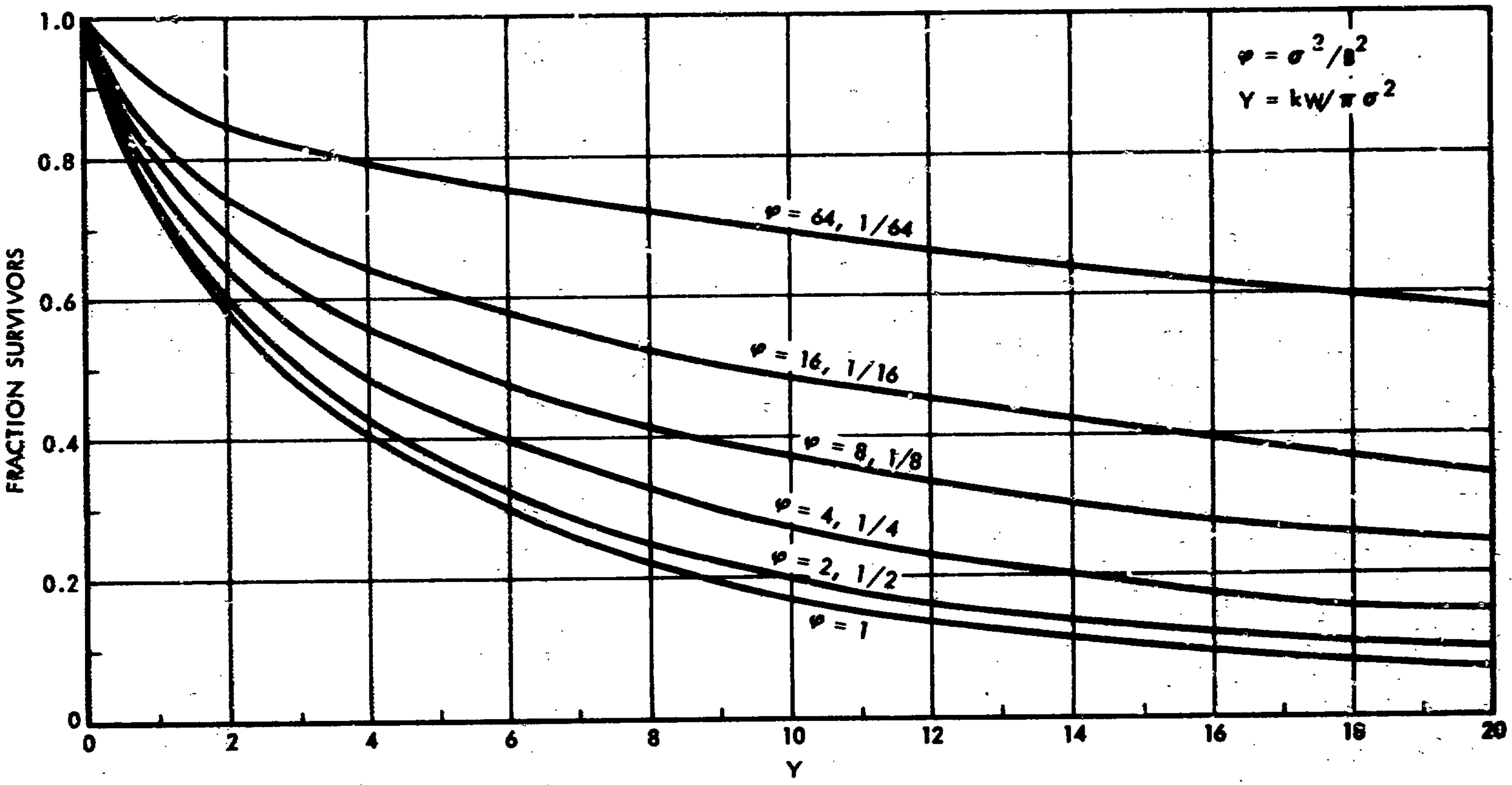

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Figure 6. FRACTION SURVIVORS OF GAUSSIAN POPULATION WITH STANDARD DEVIATION β WITH TARGETING AGAINST GAUSSIAN POPULATION WITH STANDARD DEVIATION σ WITH WEAPONS NORMALIZED FOR POPULATION-- $\varphi = \sigma^2/\beta^2$, $Y = kW/\pi\sigma^2$

Now consider another evaluation function,

$$F_c(\omega) = 1 - e^{-k_c \omega} .$$

This function will be used for two cases:

Case I: The actual vulnerability is k_c and the effect of the attacker using an incorrect vulnerability is of interest;

Case II: The k_c represents a casualty rate and we wish to determine injuries (casualties minus fatalities).

In either case, define

$$\xi = k_c/k \cdot \qquad (54)$$

For fatalities, when $F(\omega)$ is used directly with k, the square root law derivation is used, leading to

$$H/V_0 = 1 - \exp(-\sqrt{X})(1 + \sqrt{X}) ,$$

where X is defined as $kW/\pi\sigma^2$.

Call H_c the value obtained using k_c.

$$H_c = \int_0^{r_c} F_c(\omega)\, V(r)\, dA . \qquad (55)$$

As before, the integration could be directly performed using

$$F_c = 1 - e^{-k_c \omega} = 1 - \gamma^{\xi} \exp(\xi\, r^2/2\sigma^2) .$$

However, the following change of variables leads to less algebra. Let

$$dV = -V\, dA/2\pi\sigma^2 , \qquad (56)$$

so

$$H_c = 2\pi\sigma^2 \int_{\lambda/k}^{V_0/2\pi\sigma^2} F_c(\omega)\, dV \; . \tag{57}$$

From the definition of k_c, and substituting for ω,

$$F_c = 1 - e^{-k_c\omega} = 1 - (\lambda/kV)^{\xi} \; , \tag{58}$$

so

$$H_c = 2\pi\sigma^2 \int_{\lambda/k}^{V_0/2\pi\sigma^2} [1 - (\lambda/kV)^{\xi}]\, dV \; .$$

Now let $y = kV/\lambda$ to give the simple expression

$$H_c/V_0 = \gamma \int_1^{1/\gamma} (1 - y^{-\xi})\, dy \; .$$

From which

$$H_c/V_0 = 1 - \gamma + \frac{\gamma - \gamma^{\xi}}{1 - \xi} = 1 + \gamma \frac{\xi}{1 - \xi} - \frac{\gamma^{\xi}}{1 - \xi} \; . \tag{59}$$

For Case I, we wish values of H_c directly. Recalling

$$\gamma = \exp(-\sqrt{X}) \; ,$$

and calling

$$S_c = 1 - H_c/V_0 \; , \tag{60}$$

$$S_c = \frac{\xi}{\xi - 1} \exp(-\sqrt{X}) - \frac{\exp(-\xi\sqrt{X})}{\xi - 1} \; . \tag{61}$$

Here the weapon usage is normalized by the original attack. Values of S_c are given in Figure 7. Normalizing by the actual vulnerability gives

$$Y = k_c W/\pi\sigma^2 = \xi X \; . \tag{62}$$

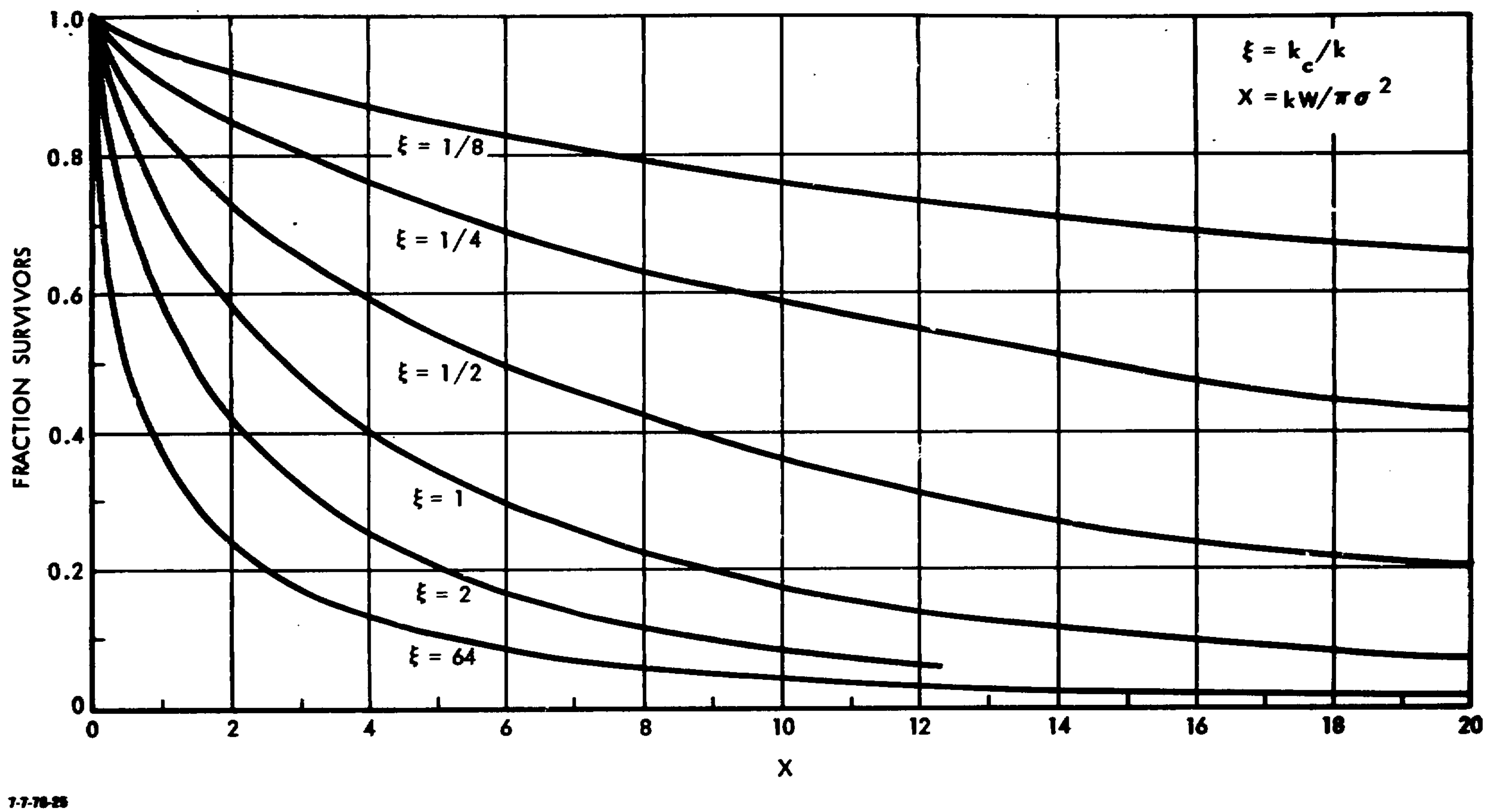


Figure 7. FRACTION SURVIVORS AS A FRACTION OF WEAPON USAGE FOR VULNERABILITY INDEX k_c AND TARGET k WITH WEAPONS NORMALIZED FOR ATTACK--$\xi = k_c/k$, $X = kW/\pi\sigma^2$

Then

$$S_c = \frac{\xi}{\xi - 1} \exp(-\sqrt{Y/\xi}) - \frac{1}{\xi - 1} \exp(-\sqrt{Y\xi}) . \quad (63)$$

Replacing ξ by $1/\xi$ in this formula leaves survivors unaffected, which implies that either an underestimation or an overestimation of vulnerability will give the same result. Moreover, this formula is identical to that for survivors of two Gaussian populations of different dispersions in the previous section with φ replaced by ξ. The curve of survivors in that case, Figure 6, also applies here.

For Case II, call injuries the difference between fatalities and casualties, so

$$I = H_c/V_0 - H/V_0 . \quad (64)$$

Then since $H/V_0 = 1 - \gamma - \gamma \ln(1/\gamma)$, we have, from Equation (59),

$$I = \frac{\gamma - \gamma^{\xi}}{1 - \xi} + \gamma \ln(1/\gamma). \quad (65)$$

Thus

$$I = \frac{\exp(-\sqrt{X}) - \exp(-\xi\sqrt{X})}{1 - \xi} + \sqrt{X} \exp(-\sqrt{X}) ,$$

or

$$I = \exp(-\sqrt{X}) \frac{1 - \exp((1 - \xi)\sqrt{X})}{1 - \xi} + \sqrt{X} . \quad (66)$$

Injuries are presented in Figure 8 for values of ξ greater than 1. In this Figure, the attacker is attempting to maximize fatalities. If the attacker is attempting to maximize total casualties--injuries plus fatalities--then values of ξ are less than 1; i.e., the vulnerability of the evaluated population is less than assumed by the attacker. The injuries in this case are presented in Figure 9.

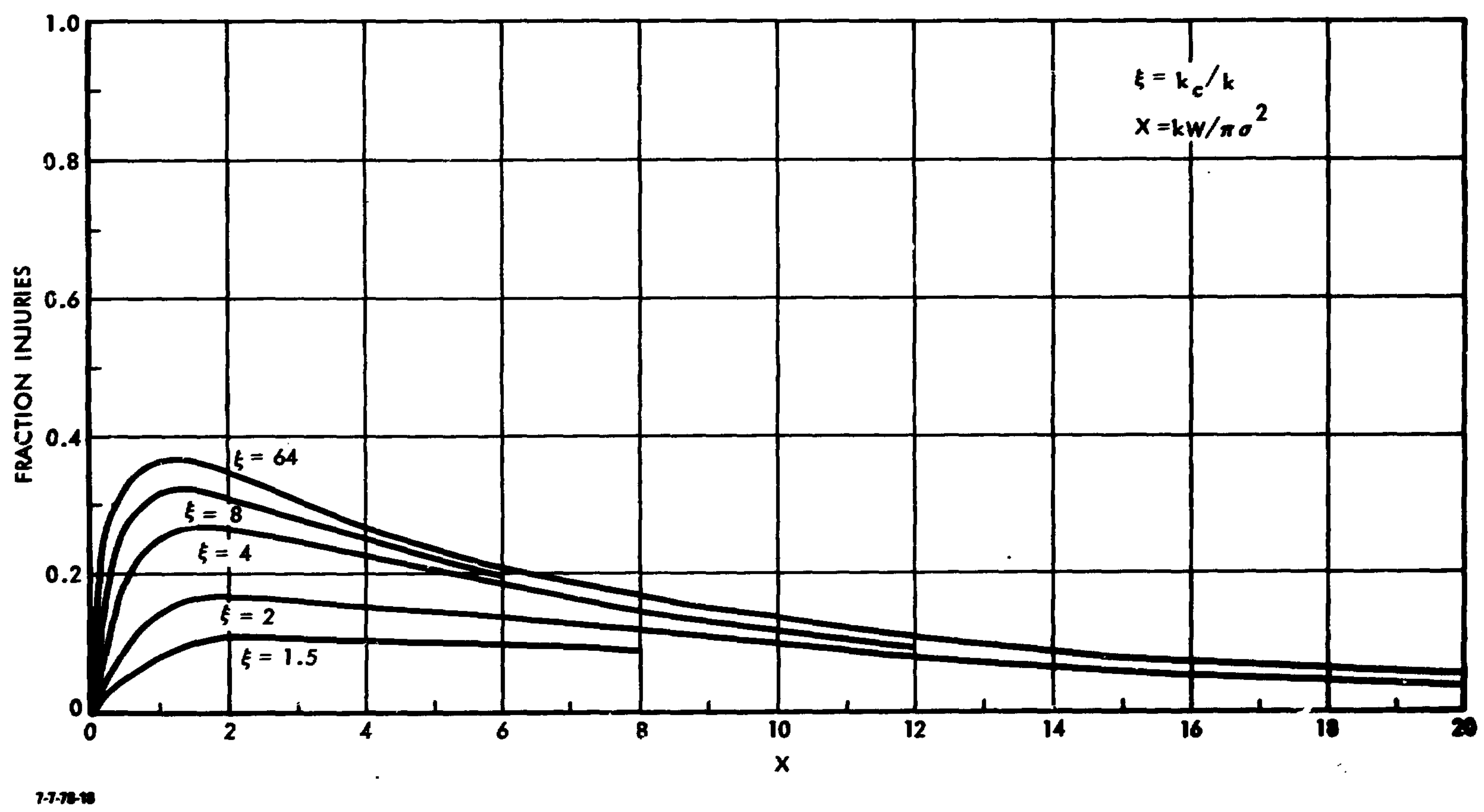


Figure 8. INJURIES AS A FUNCTION OF WEAPON USAGE WHEN ATTACKING FOR FATALITIES--$\xi = k_c/k$, $X = kW/\pi\sigma^2$

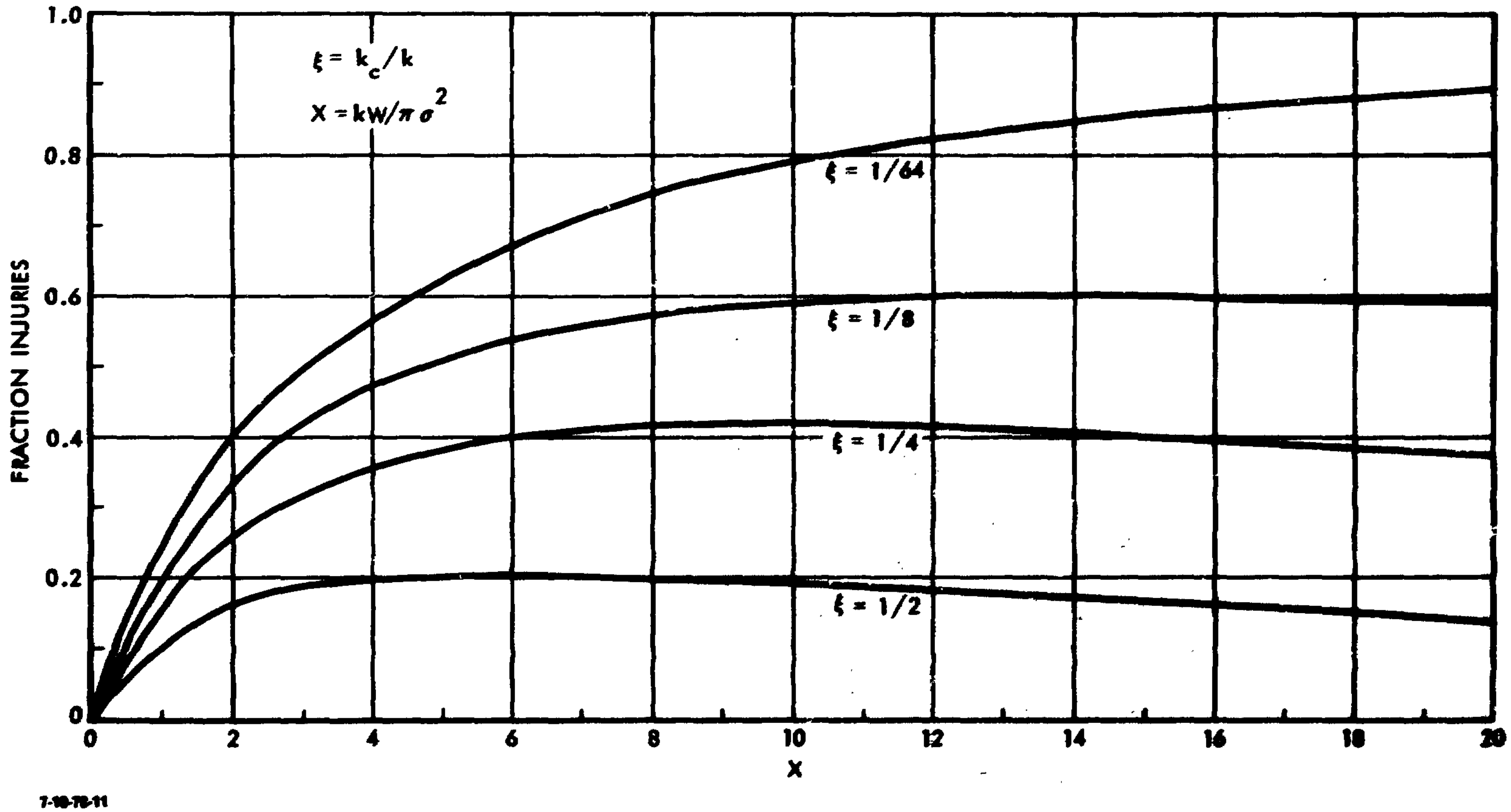
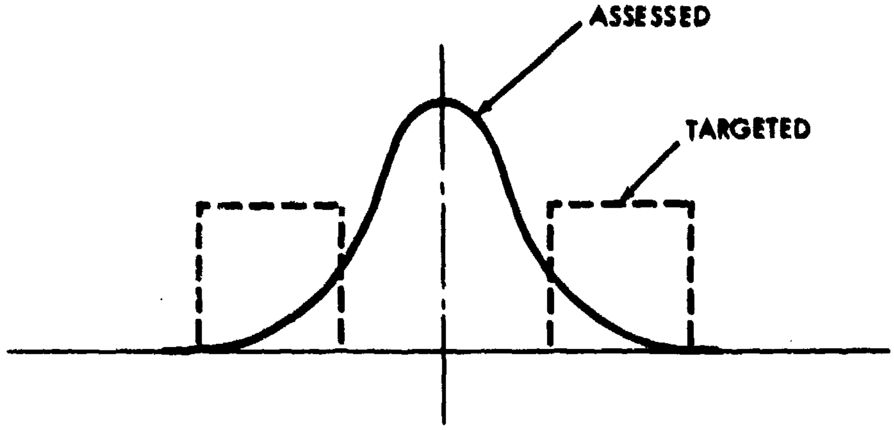


Figure 9. INJURIES AS A FUNCTION OF WEAPONS USAGE WHEN ATTACKING FOR TOTAL CASUALTIES--$\xi = k_c/k$, $X = kW/\pi\sigma^2$

F. SURVIVORS FROM A GAUSSIAN POPULATION TARGETED AS A UNIFORM POPULATION IN AN ANNULUS



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Assume a population, V_0, is Gaussian distributed with standard deviation σ so

$$V(r) = V_0/2\pi\sigma^2 \exp(-r^2/2\sigma^2) \ .$$

The targeting is assumed to be against a uniform ring with inner radius L and outer radius H. In order to normalize the evaluation in a way comparable to other situations, a "center of gravity" and "standard deviation" of the ring is defined by

$$m = 1/A \int_L^H r \cdot 2\pi r \, dr \ , \tag{67}$$

where A is the ring area = $\pi(H^2 - L^2)$, and

$$B^2 = 1/A \int_L^H (r - m)^2 \, 2\pi r \, dr \ . \tag{68}$$

This yields

$$m = \frac{2}{3} \frac{H^3 - L^3}{H^2 - L^2} , \tag{69}$$

$$B^2 = \frac{1}{2} \frac{H^4 - L^4}{H^2 - L^2} - m^2 . \tag{70}$$

The expression for B is a measure of dispersion about the "center strip" of an annulus. Thus, as the inner radius of the annulus approaches 0, B does not approach the value computed for a disk which is about the center. In fact, with L = 0, we have

$$B_{ring} = 1/3 \, B_{disk} .$$

There is no simple solution for H and L in terms of m and B, but a simple Newton's method type of procedure will allow for quickly determining numerical values. Figure 10 shows m and B as a function of H and L. As the thickness of the ring becomes small compared to a radius, the ring curvature is insignificant in determining m and B, so that we have approximately

$$H \approx m + \sqrt{3B} , \qquad B/m << 1 ,$$

$$L \approx m - \sqrt{3B} , \qquad B/m << 1 .$$

The departures of the contours of constant m and B in Figure 10 from straight lines where L/H is small indicate departures from this approximation.

The ratio of target to actual standard deviation, B/σ, we shall call ρ, and the ratio of ring center to standard deviation m/σ, we shall call η. At small L, the φ used previously is related to ρ by $\varphi = 1/9\rho^2$ if β is substituted for 3B.

The total weapon usage W is given by weapon density times ring area, or

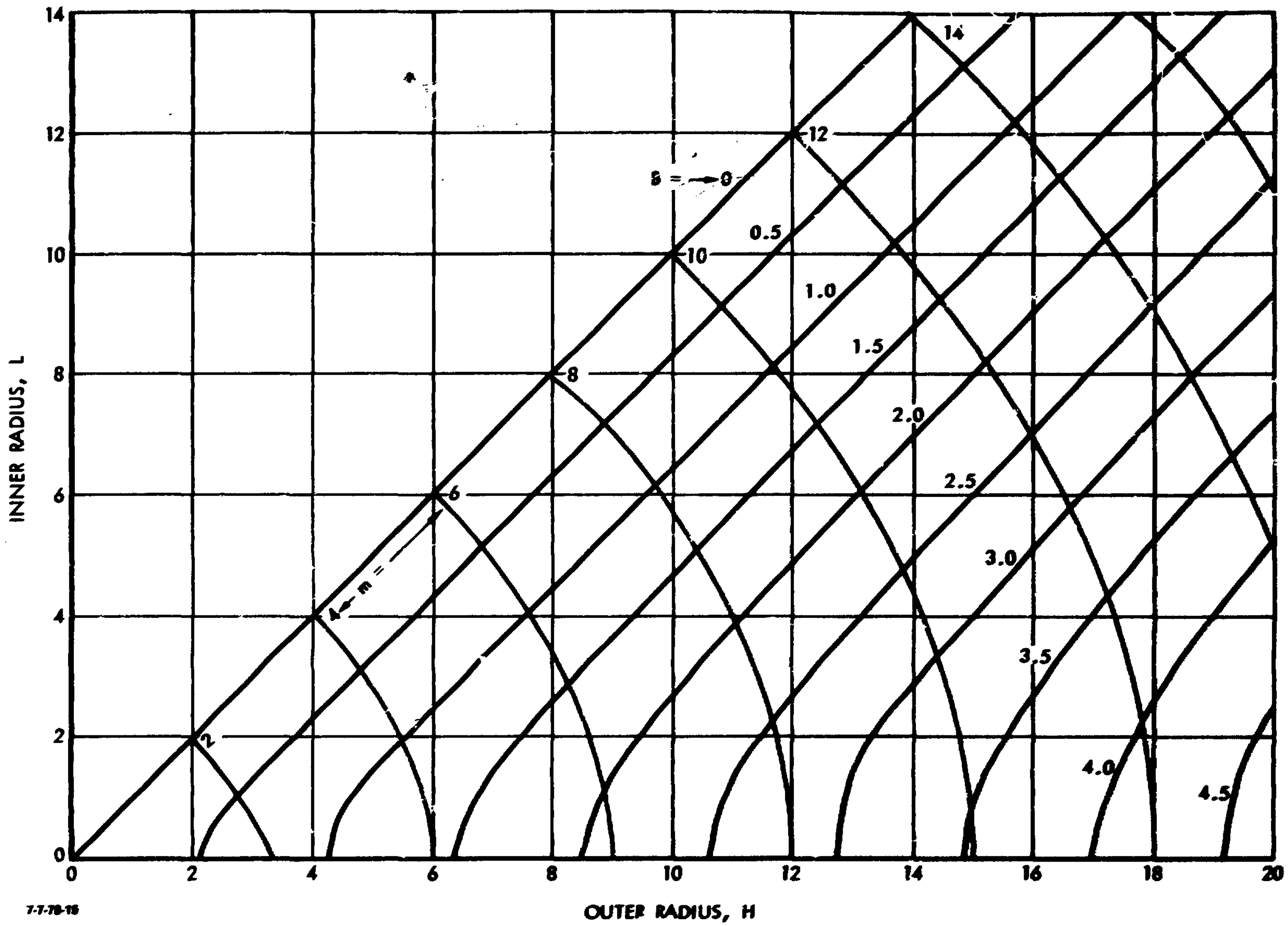


Figure 10. MEAN AND STANDARD DEVIATION OF ANNULUS AS A FUNCTION OF INNER AND OUTER RADIUS

$$W = \pi(H^2 - L^2)\omega \ .$$

Call X the weapon usage normalized to the target, or

$$X = kW/\pi B^2 = k\omega(H^2 - L^2)/B^2 \ . \tag{71}$$

Call Y the weapon usage normalized to the assessed population, or

$$Y = kW/\pi B^2 = k\omega(H^2 - L^2)/\sigma^2 \ . \tag{72}$$

The succeeding calculations will be in terms of Y but could be readily converted to X by changing σ to B. The calculations will be done explicitly in terms of H and L, which are functions of m and B. As the expression for Y shows, these functions can be considered as normalized by σ; i.e., the expressions are in terms of H/σ, L/σ, ρ and η.

The fraction survivors k is given by

$$F = 1 - \exp(-k\omega) = 1 - \exp\left(- \frac{Y}{H^2/\sigma^2 - L^2/\sigma^2}\right) \ . \tag{73}$$

The value destroyed is given by

$$H = \int_L^H F \cdot V \, dA \ ;$$

$$= \int_L^H \left(1 - \exp\left[- \frac{Y}{H^2/\sigma^2 - L^2/\sigma^2}\right]\right) V_0/2\pi\sigma^2 \exp(-r^2/2\sigma^2) 2\pi r \, dr \ .$$

Integrating, and letting the fraction of survivors, $S = 1 - H/V_0$, gives

$$S = 1 - \left[1 - \exp\left(- \frac{Y}{(H/\sigma^2) - (L/\sigma)^2}\right)\right] \left[\exp\left(- \frac{L^2}{2\sigma^2}\right) - \exp\left(- \frac{H^2}{2\sigma^2}\right)\right]. \tag{74}$$

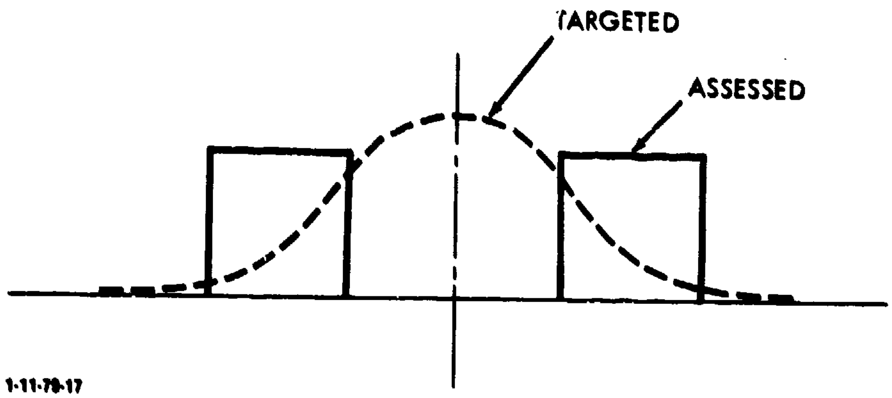
(A similar formula based on S is attained by substituting X for Y and B for σ.)

Fraction survivors are indicated in Figure 11 for the annulus center, m, at a constant value with varying annulus width, B; and in Figure 12 for the annulus width, B, constant and varying distance of the center m. In these figures a curve for L = 0 is given which reduces to the disk calculations presented earlier.

When L/H is near 1 and the approximate formulas for H and L can be used, we have

$$S \approx 1 - [1 - \exp(-Y/(4\sqrt{3}\eta\rho))][\exp(-(\eta - \sqrt{3}\rho)^2/2) - \exp(-(\eta + \sqrt{3}\rho)^2/2)] \quad (75)$$

G. SURVIVORS FROM A POPULATION IN AN ANNULUS TARGETED AS A GAUSSIAN POPULATION



Assume a population is of constant density in a ring of inner diameter L and outer diameter H. Let m and σ be the mean ring distance and σ the standard deviation. The relations between H and L and m and σ are the same as in the previous section, with σ replaced by B. The targeting is assumed against a Gaussian target distributed

$$V = V_T/2\pi\beta^2 \exp(-r^2/2\beta^2) .$$

As in the square root law derivation, the weapon usage is

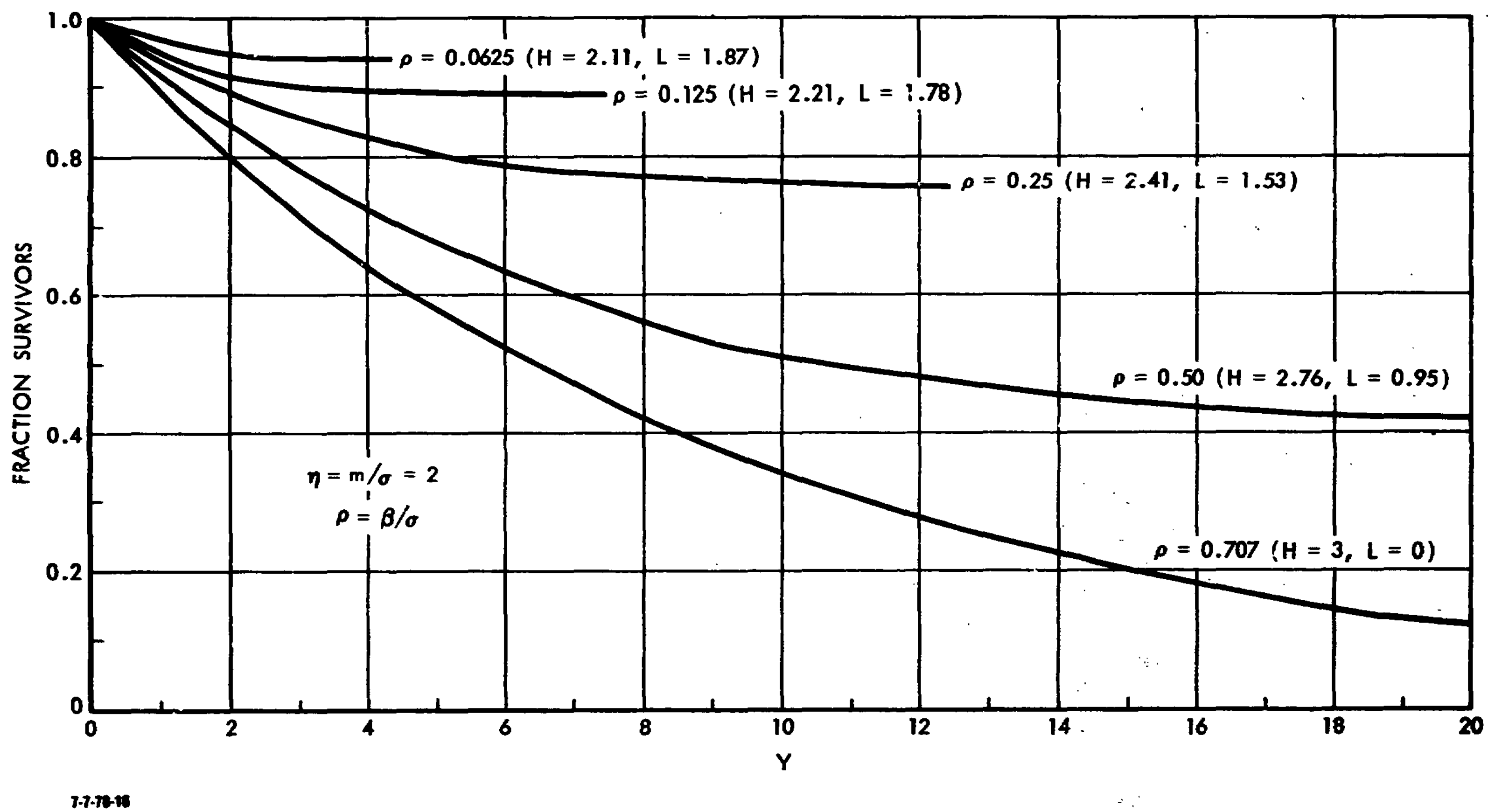


Figure 11. SURVIVORS FROM A GAUSSIAN POPULATION TARGETED AS AN ANNULUS WITH CENTER A CONSTANT DISTANCE--$\eta = m/\sigma = 2$, $\rho = \beta/\sigma$

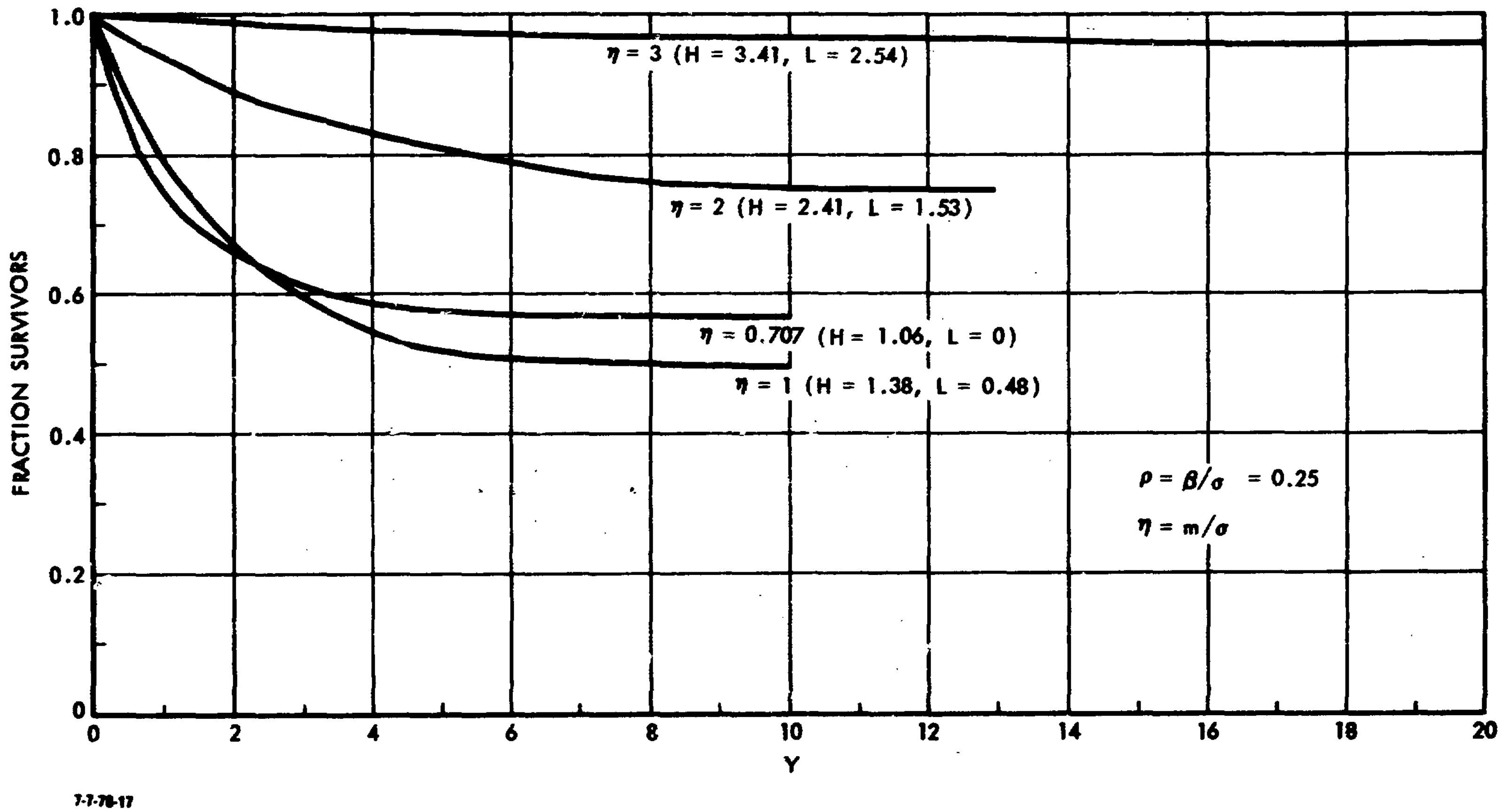


Figure 12. SURVIVORS FROM A GAUSSIAN POPULATION TARGETED AS AN ANNULUS WITH CONSTANT WIDTH--$\rho = \beta/\sigma = 0.25$, $\eta = m/\sigma$

$$X = kW/\pi\beta^2 = \ln^2(1/\gamma) ,$$

with

$$\gamma = 2\pi\beta^2\lambda/kV_0 .$$

The fraction damage F is

$$F = 1 - \gamma \exp(r^2/2\beta^2) ,$$

and extends to a distance r_c where r_c solves

$$\gamma = \exp(-r_c^2/2\beta^2) .$$

For r_c less than L there is no damage and the fraction survivors is 1. Otherwise, for damage

$$H = \int_L^J [1 - \gamma \exp(r^2/2\beta^2)] [V_0/\pi(H^2 - L^2)] 2\pi r \, dr , \tag{76}$$

where

$$J = \min(r_c,H) . \tag{77}$$

This readily gives

$$S = 1 - H/V_0 = 1 - \{\frac{J^2 - L^2}{H^2 - L^2} - 2 \exp \frac{(- \sqrt{X})\beta^2}{H^2 - L^2} [\exp(J^2/2\beta^2)$$

$$- \exp(L^2/2\beta^2)]\} .$$

If $r_c > H$ this simplifies to

$$S = 2 \exp \frac{(- \sqrt{X})\beta^2}{H^2 - L^2} [\exp(H^2/2\beta^2) - \exp(L^2/2\beta^2)] . \tag{78}$$

For $r_c < H$ this can be written

$$S = 1 - \frac{2\beta^2 \sqrt{X} - L^2}{H^2 - L^2} + \frac{2\beta^2}{H^2 - L^2} - 2 \exp(- \sqrt{X}) \cdot \beta^2 \cdot \exp(L^2/2\beta^2). \tag{79}$$

In terms of the assessed population, the normalization becomes

$$Y = kW/\pi B^2 = X/\psi \tag{80}$$

where $\psi = B^2/\beta^2$. The fraction of survivors as a function of Y is shown in Figure 13 for an annulus of constant ratio of outer to inner diameter of 2.9. When the targeted population is not dispersed enough, the weapons are concentrated in the hole in the annulus. As the standard deviation of the targeted population is increased, the targeting becomes closer to optimal targeting until finally the weapons are too dispersed and the targeting effectiveness drops off.

H. AN ALTERNATIVE SQUARE ROOT LAW DERIVATION

In this derivation, the geometric context of the square root law will be emphasized. As before, let

$$V(r) = V_0/2\pi\alpha^2 \exp(-r^2/2\alpha^2) \; .$$

Let

$$\delta^2 = 2\alpha^2 \; ,$$

so

$$\delta = \sqrt{2}\alpha \; . \tag{81}$$

Then

$$V(r) = V_0/\pi\delta^2 \exp(-r^2/\delta^2) \; .$$

Now call

$$\pi\delta^2 = A_T \; , \tag{82}$$

so A_T is the circle which contains 1 - 1/e, or 63 percent of the population.

Let

$$A = \pi r^2 \; .$$

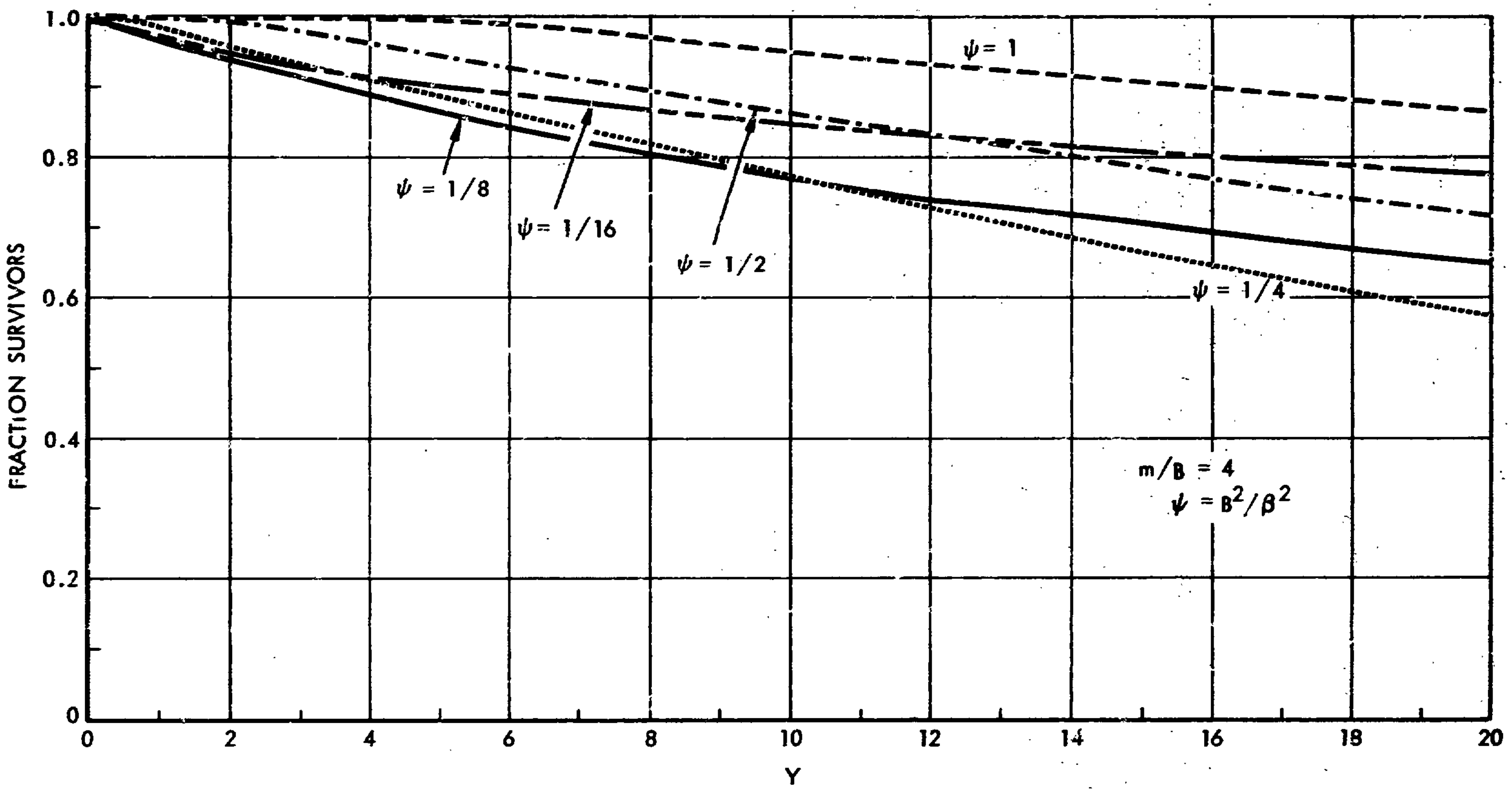


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Figure 13. SURVIVORS FROM A UNIFORM ANNULUS POPULATION TARGETED AS A GAUSSIAN POPULATION FOR H/L = 2.9

Then

$$V(r) = V_0/A_T \exp(-A/A_T) \ . \tag{83}$$

As before, let

$$F(\omega) = 1 - e^{-k\omega} \ .$$

The optimal allocation yields the requirement

$$e^{-k\omega} = \lambda/kV, \quad \text{for } \lambda/kV \leq 1 \ .$$

Now let $s(\omega)$ denote the fraction of survivors, locally, so

$$s(\omega) = e^{-k\omega} = \lambda/kV \ .$$

Call $G = sV$ the survivors per unit area. Then

$$G = V \cdot \lambda/kV = \lambda/k \ . \tag{84}$$

Thus the survivors per unit area are a constant for the area attacked.

The attack continues to a radius where $\lambda/kV = 1$. Call the area enclosed A_C. At this radius the target value equals the survivors per unit area. Thus,

$$V/G = 1 \ ,$$

so

$$V_0/GA_T \exp(-A_C/A_T) = 1,$$

or

$$\ln(V_0/GA_T) = A_C/A_T \ . \tag{85}$$

Define, as before, a dimensionless weapon usage by

$$X = kW/\pi\alpha^2 = 2/A_T \int_0^{A_C} k\omega \, dA \ . \tag{86}$$

Now, since

$$e^{-k\omega} = G/V \ ,$$

$$k\omega = \ln(V/G) = \ln(V_0/GA_T) - A/A_T \ .$$

Thus

$$X = 2/A_T \int_0^{A_C} (\ln(V_0/GA_T) - A/A_T)\, dA$$

$$= 2/A_T \ln(V_0/GA_T)A_C - 2/A_T^2\, A_C^2/2$$

$$= 2\, A_C/A_T\, (\ln(V_0/GA_T) - 1/2\, A_C/A_T) \ .$$

Since $A_C/A_T = \ln(V_0/GA_T)$,

$$X = (A_C/A_T)^2 \ , \tag{87}$$

or alternatively,

$$X = \ln^2(V_0/GA_T) \ .$$

Divide survivors into the area in the circle A_C and the rest of the plane

$$S = S_1 + S_2 \ .$$

$$S_1 = GA_C \ .$$

$$S_2 = \int_{A_C}^{\infty} 1 \cdot V\, dA = \int_{A_C}^{\infty} V_0/A_T \exp(-A/A_T)\, dA \ .$$

$$= V_0 \exp(-A_C/A_T).$$

$$S = GA_C + V_0 \exp(-A_C/A_T).$$

Normalizing,

$$S/V_0 = GA_C/V_0 + \exp(-A_C/A_T) \ . \tag{88}$$

Now S will be computed both in terms of G and A_C.

First, geometrically:

$$S/V_0 = GA_T/V_0 \cdot A_C/A_T + \exp(-A_C/A_T) .$$

Now,

$$\ln(GA_T/V_0) = -A_C/A_T ,$$

so

$$GA_T/V_0 = \exp(-A_C/A_T) ,$$

$$S/V_0 = \exp(-A_C/A_T)\ A_C/A_T + \exp(-A_C/A_T) ,$$

$$= \exp(-A_C/A_T)(A_C/A_T + 1) . \tag{89}$$

And since

$$A_C/A_T = \sqrt{X} ,$$

$$S/V_0 = \exp(-\sqrt{X})(\sqrt{X} + 1) .$$

Now in terms of survivors

$$\frac{S}{V_0} = \frac{GA_T}{V_0} \ln\left(\frac{V_0}{GA_T}\right) + \frac{GA_T}{V_0}$$

$$= GA_T/V_0 \left(\ln(V_0/GA_T) + 1\right). \tag{90}$$

Now

$$GA_T/V_0 = \exp(-\sqrt{X}) ,$$

so $$S/V_0 = \exp(-\sqrt{X})(\sqrt{X} + 1) .$$

I. A GENERALIZATION OF THE SQUARE ROOT DAMAGE LAW

Suppose value, as a function of area covered, is an arbitrary decreasing function. Then the use of weapcn density can still be used for a more general form of the square root law.

Let

$$V(A) = \frac{V_0}{A_T} f\left(\frac{A}{A_T}\right) \tag{91}$$

be the value per unit area where value density is a decreasing function of area; i.e.,

$$\frac{df(x)}{dx} < 0 \ . \tag{92}$$

Let

$$g(x) = \int_0^x f(t)dt \ . \tag{93}$$

Now

$$V_0 = \int_0^\infty V(A)dA = \int_0^\infty \frac{V_0}{A_T} f\left(\frac{A}{A_T}\right)dA$$

$$= V_0 \int_0^\infty f(A)dA \ .$$

Thus we require

$$g(\infty) = 1 \ . \tag{94}$$

Moreover, to normalize distance in a fashion comparable to the Gaussian case, set

$$g(1) = 1 - \frac{1}{e} \approx 0.63 \ . \tag{95}$$

The damage, as a function of weapon density, is still

$$F(\omega) = 1 - e^{-k\omega} \ .$$

The optimization of an attack yields

$$\frac{\partial \ V(A)F(\omega)}{\partial \omega} = \frac{V}{K} e^{-k\omega} = \lambda \ ,$$

so

$$e^{-k\omega} = \frac{\lambda}{kV} \text{ for } \frac{\lambda}{kV} \leq 1 \; .$$

Call s the local fraction of survivors and G the local number of survivors per unit area. Then

$$G = Vs = V(1-f) = Ve^{-k\omega} = V\frac{\lambda}{kV} = \frac{\lambda}{k} \; .$$

Thus G is a constant in the area attacked. The attack is made out to a radius where

$$\frac{\lambda}{kV} = 1$$

or

$$V = G \; .$$

Call A_C the area attacked. Then

$$f\left(\frac{A_C}{A_T}\right) = \frac{GA_T}{V_0} \; . \tag{96}$$

The weapon usage W is given by

$$W = \int_0^{A_C} \omega dA \; .$$

Let X be the normalized weapon usage, as before, so

$$X = \frac{2kW}{A_T} = \frac{2}{A_T} \int_0^{A_C} k\omega dA \; . \tag{97}$$

Now

$$e^{-k\omega} = \frac{G}{V} \, ,$$

so

$$k\omega = \ln\left(\frac{V}{G}\right) = \ln\left(\frac{V_0}{GA_T} \quad f\left(\frac{A}{A_T}\right)\right) \; .$$

Then

$$X = \frac{2}{A_T}\int_0^{A_C} \ln\left(\frac{V_0}{GA_T}\right) f\left(\frac{A}{A_T}\right) dA$$

$$= 2\int_0^{\frac{A_C}{A_T}} \ln\left(\frac{V_0}{GA_T}\right) f\left(\frac{A}{A_T}\right) d\left(\frac{A}{A_T}\right)$$

$$= 2\frac{A_C}{A_T} \ln\left(\frac{V_0}{GA_T}\right) + 2\int_0^{\frac{A_C}{A_T}} \ln(f(x))dx \ . \quad (98)$$

Now define

$$h(x) = \int_0^x \ln(f(t))dt \ . \quad (99)$$

Then

$$X = 2\frac{A_C}{A_T} \ln\left(\frac{V_0}{GA_T}\right) + 2h\left(\frac{A_C}{A_T}\right) . \quad (100)$$

The total survivors is the sum S_1 of the survivors in the area attacked and S_2 in the unattacked area with

$$S_1 = A_C G$$

$$S_2 = V_0\left(1-g\left(\frac{A_C}{A_T}\right)\right) .$$

The fraction of survivors is then

$$\frac{S}{V_0} = \frac{A_C}{A_T}\frac{GA_T}{V_0} + 1 - g\left(\frac{A_C}{A_T}\right) . \tag{101}$$

J. AN APPLICATION OF THE GENERALIZED DAMAGE LAW TO VALUE A LINEAR FUNCTION OF AREA

Let value be a linear function of area, say of the form

$$V(A) = \begin{cases} V_0 \dfrac{2}{A_F}\left(1 - \dfrac{A}{A_F}\right) , & A \leq A_F \\ 0 , & A > A_F . \end{cases} \tag{102}$$

For a circularly symmetric value function, value would decrease quadratically with distance. After normalizing with the area A_T, we have

$$V = \begin{cases} \dfrac{V_0}{A_T} 2\tilde{e}\left(1 - \tilde{e}\dfrac{A}{A_T}\right) , & \dfrac{A}{A_T} \leq \dfrac{1}{\tilde{e}} \\ 0 , & \dfrac{A}{A_T} > \dfrac{1}{\tilde{e}} , \end{cases} \tag{103}$$

where $\tilde{e} = 1 - \sqrt{1/e} \approx 0.39346934$. (104)

Then

$$g(x) = \begin{cases} 2\tilde{e}x - \tilde{e}^2x^2 , & x \leq \dfrac{1}{\tilde{e}} \\ 1 , & x > \dfrac{1}{\tilde{e}} , \end{cases}$$

and it is readily seen that $g(1) = 1 - \frac{1}{e}$, as desired for normalization. Moreover, in terms of the original triangle,

$$A_T = \tilde{e}A_F .$$

For $h(x)$ we have

$$h(x) = \left(x - \frac{1}{\tilde{e}}\right)\log(2\tilde{e} - 2\tilde{e}^2 x) - x + \frac{1}{\tilde{e}} \log(2\tilde{e}) \ . \qquad (105)$$

Now suppose a value of G (survivors/unit area) is given. Then with A_C defined as the area under attack, call $x_c = A_C/A_T$. Then

$$x_c = \frac{1}{\tilde{e}} - \frac{GA_T}{V_0}\frac{1}{2\tilde{e}^2} \ .$$

Now from

$$X = 2x_c \ \ln\left(\frac{V_0}{GA_T}\right) + 2h(x_c) \ ,$$

we have

$$X = \frac{2}{\tilde{e}} \ln\left(\frac{V_0}{GA_T}\right) + \frac{1}{\tilde{e}^2}\frac{GA_T}{V_0} + \frac{2}{\tilde{e}}(\log 2\tilde{e} - 1) \ , \qquad (106)$$

and from

$$\frac{S}{V_0} = x_c\left(\frac{GA_T}{V_0}\right) + 1 - g(x_c) \ ,$$

we have

$$\frac{S}{V_0} = \frac{1}{\tilde{e}}\frac{GA_T}{V_0} - \frac{1}{4\tilde{e}^2}\left(\frac{GA_T}{V_0}\right)^2 \ . \qquad (107)$$

Numerically, this gives

$$X = 5.0829881 \ \ln\left(\frac{V_0}{GA_T}\right) + 6.4591921\left(\frac{GA_T}{V_0}\right) - 6.3008972$$

$$S/V_0 = 2.5414940\left(\frac{GA_T}{V_0}\right) - 1.6147980\left(\frac{GA_T}{V_0}\right)^2 \ .$$

In this case no explicit function $\frac{S}{V_0}(x)$ is presented. Solutions must be obtained in terms of the parameter $\frac{GA_T}{V_0}$. The valid range of area attacked x_c is

$$0 \leq x_c \leq \frac{1}{\tilde{e}} \ ,$$

which gives

$$2\bar{a} = 0.79693868 \geq \frac{QA_T}{V_0} \geq 0 \ .$$

The survivors, as a function of attack intensity, is presented in Figure 14. The square root law, with Gaussian targets, is included for comparison. As can be seen, practically no difference occurs between the two curves until the survivors are under 50 percent. Then the larger amounts of area at low value density in the Gaussian targets begin to significantly affect the attack effectiveness. In Figure 15, the survivors, attack size, and several other parameters are presented as a function of the area attacked. As is clear in the Figure, not until almost all the area is attacked does the weapons usage become relatively large.

K. AN APPLICATION OF THE GENERALIZED DAMAGE LAW TO VALUE AN EXPONENTIAL FUNCTION OF AREA TO A POWER

Let the value as a function of area be given by

$$V(A) = V_0\bar{\alpha}\exp\{-(\beta A)^n\} \ , \qquad 0 < n < \infty \ . \tag{108}$$

The area A is πr^2, so this can be written

$$V(A) = V_0\bar{\alpha}\exp\left(-\{\beta\pi r^2\}^n\right) \ .$$

For n = 1 the standard Gaussian shaped curve is obtained. For $n = \frac{1}{2}$ an exponential decay is obtained. $\bar{\alpha}$ and β are parameters whose values will be determined by normalizing the value function. The damage will be calculated using the second alternative derivation. To normalize, call

$$f(x) = \bar{\alpha}\exp\left(-\{\beta A\}^n\right) \ , \tag{109}$$

and

$$g(x) = \int_0^x f(t)\,dt \ . \tag{110}$$

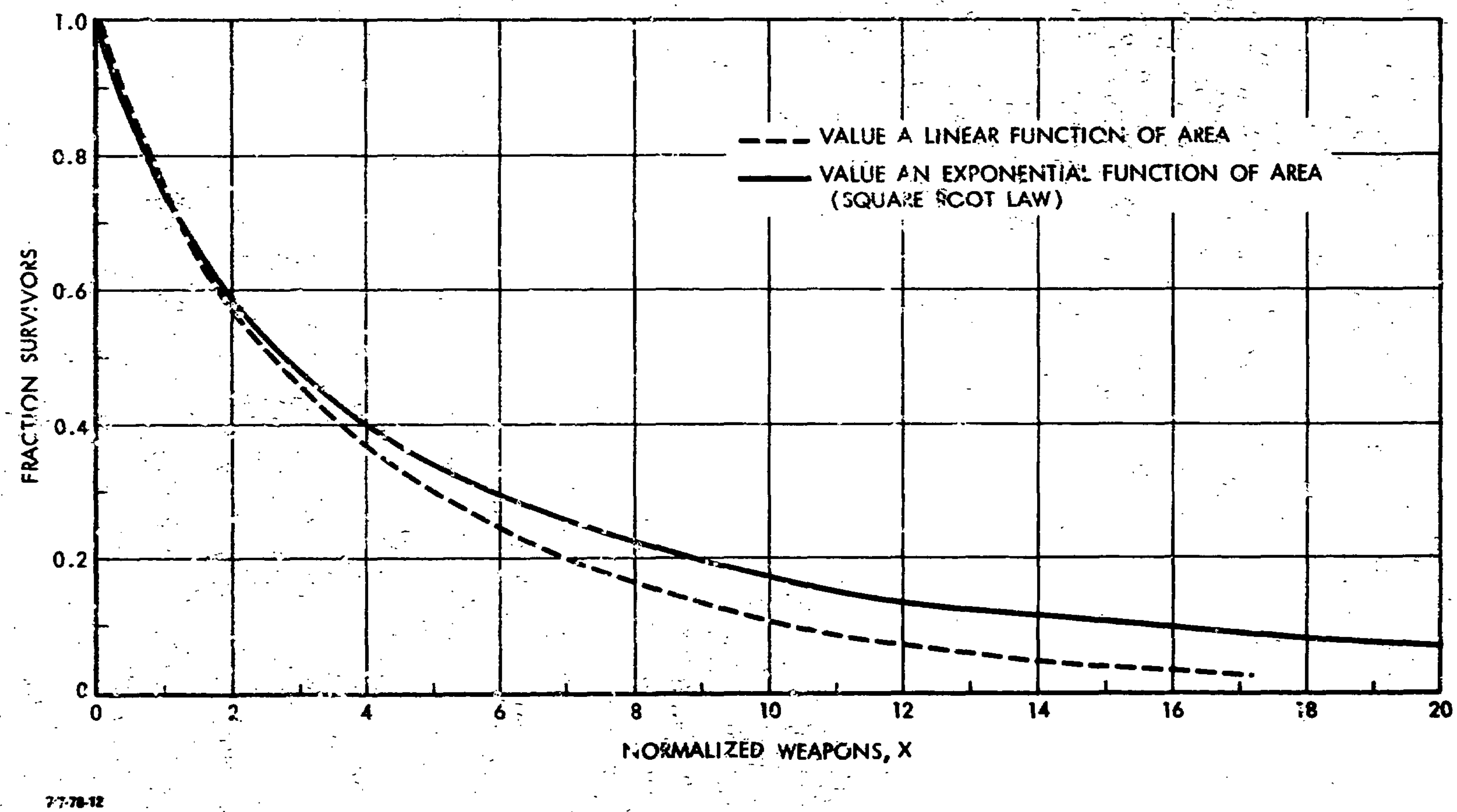

Figure 14. FRACTION SURVIVORS AS A FUNCTION OF NORMALIZED WEAPONS WITH VALUE A LINEAR FUNCTION OF AREA

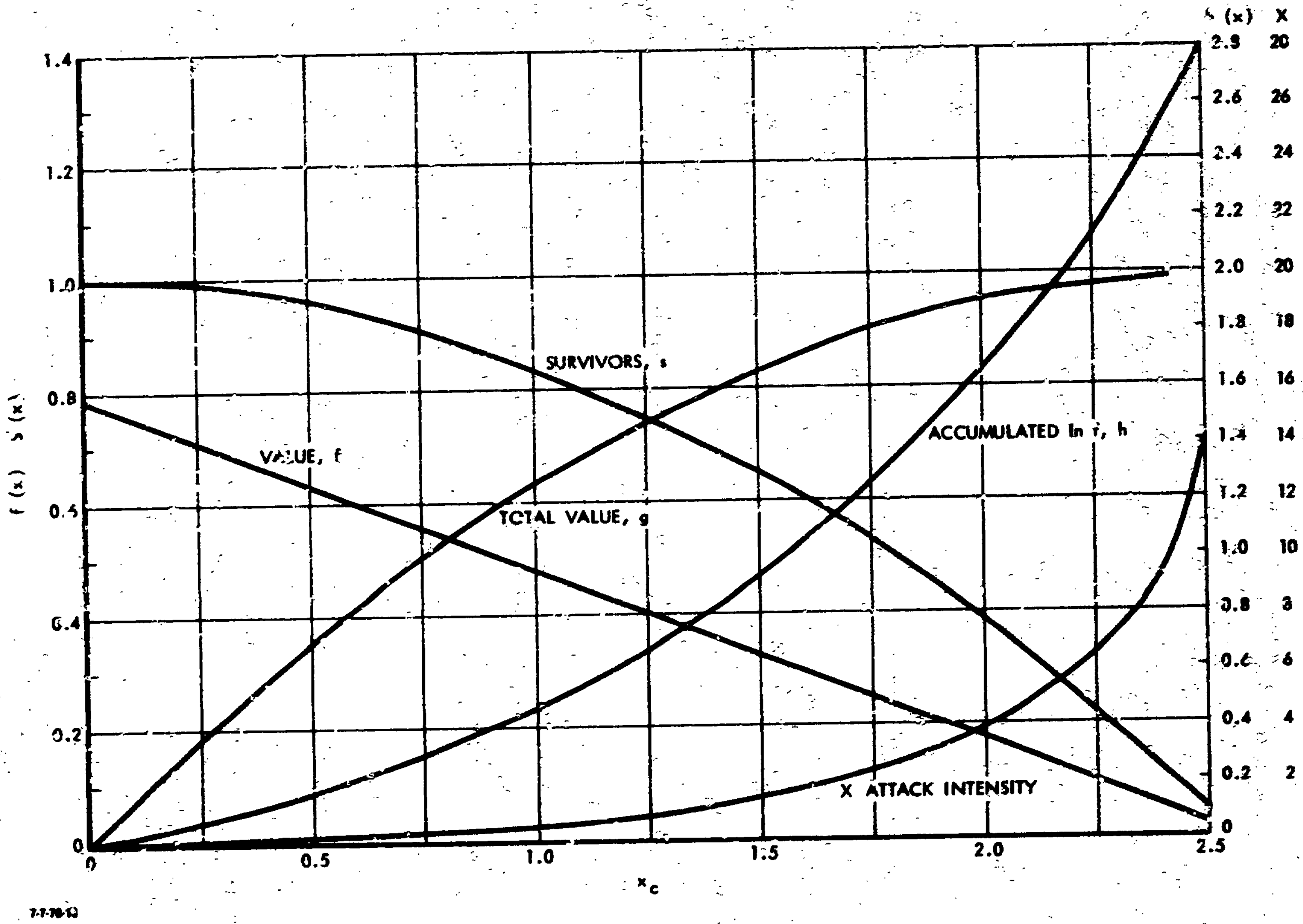

Figure 15. VARIATION OF VALUE, ACCUMULATED VALUE, ACCUMULATED LOGARITHM OF VALUE, SURVIVORS AND ATTACK INTENSITY WITH AREA COVERED BY ATTACK

Now for normalization we need

$$g(\infty) = 1 .$$

From the integral

$$\int_0^\infty x^n e^{-(rx)^m} dx = \frac{1}{mr^{n+1}} \Gamma\left(\frac{n+1}{m}\right) , \qquad [n+1, r, m > 0] ,$$

we have

$$g(\infty) = \frac{\bar{\alpha}}{\beta n} \Gamma\left(\frac{1}{n}\right) , \tag{111}$$

from which

$$f(x) = \frac{\beta n}{\Gamma\left(\frac{1}{n}\right)} \exp\left(-(\beta x)^n\right) , \tag{112}$$

and

$$g(x) = \frac{\beta n}{\Gamma\left(\frac{1}{n}\right)} \int_0^x \exp\left(-(\beta t)^n\right) dt = \frac{n}{\Gamma\left(\frac{1}{n}\right)} \int_0^{\beta x} \exp\left(-y^n\right) dy . \tag{113}$$

For normalization we wish

$$g(1) = 1 - \frac{1}{e} = 0.632120559 .$$

Thus β can be obtained numerically as a function of n. An approximate equation for β is

$$\beta = \begin{cases} 10^{\left(-0.225 + 1.32(\log_{10}4 - \log_{10}n)^{0.5}\right)}, & n < 4 \\ 0.57 + 0.036\log_{10}N , & n \geq 4 \end{cases} \tag{114}$$

calling $\alpha = \frac{\beta n}{\Gamma\left(\frac{1}{n}\right)}$, an approximate equation for α is (115)

$$1/\alpha = \begin{cases} 0.8 + 5.8\log_{10}n , & m \leq 0.6 \\ 0.4453(1 - m^{-1.09}) , & m > 0.6 \end{cases} \tag{116}$$

Then

$$f(x) = \alpha \exp(-(\beta x)^n) ,$$

where now α and β can be expressed as functions of n. Now for value we can also write

$$V(A) = \frac{V_0}{A_T} \alpha \exp\left(-\left(\beta \frac{A}{A_T}\right)^n\right) , \tag{117}$$

and preserve normalization. Here A_T is the area within which $1 - \frac{1}{e}$ of the population resides. Then $x = A/A_T$ becomes area expressed in units of A_T.

Now let a number of survivors per unit area G be given. We wish to find an $x_c (= A_C/A_T)$ so that

$$f(x_c) = \frac{GA_T}{V_0} = \bar{\lambda} . \tag{118}$$

Thus x_c solves

$$\alpha \exp(-(\beta x_c)^n) = \bar{\lambda} . \tag{119}$$

Since $x_c \geq 0$, then a condition on $\bar{\lambda}$ is

$$\bar{\lambda} \leq \alpha .$$

The function h(x) is defined as

$$h(x) = \int_0^x \ln(f(t))\, dt . \tag{120}$$

Here

$$h(x) = \int_0^x \ln(\alpha \exp(-(\beta t)^n))dt$$

$$= x \ln(\alpha) - \frac{\beta x^{n+1}}{n + 1} . \tag{121}$$

In particular,

$$h(x_c) = x_c \ln(\alpha) - \frac{\beta x_c^{n+1}}{n+1} . \tag{122}$$

Now

$$X = 2x_c \ln(1/\bar{\lambda}) + 2h(x_c) . \tag{123}$$

Finally, for survivors,

$$S = x_c\bar{\lambda} + 1 - g(x_c) . \tag{124}$$

The following table compares exact values of β and α[1] with values from the fitted curves.

n	β_{exact}	β_{fit}	$\Gamma(\frac{1}{n})$	$\frac{\Gamma(\frac{1}{n})}{\beta n} = \frac{1}{\alpha}$	$\ln \frac{\Gamma(\frac{1}{n})}{\beta n}$	α_{exact}	α_{fit}
0.125	27,100,000	197,840	5,040	0.0014878	-6.51044	672.1230	84.60
0.25	357.473	201	6	0.06714	-2.701	14.89	14.76
0.5	4.6224	5.0	1	0.43268	-0.83777	2.3112	2.58
0.75	1.55201	1.63	0.89298	0.7672	-2.6506	1.3035	1.18
1.0	1.00231	0.996	1.00	0.9977	-0.0023	1.0023	1.00
1.5	0.716634	0.694	1.3540	1.2596	0.23079	0.7939	0.8529
2.0	0.63778	0.623	1.7724	1.3895	0.3289	0.7197	0.7897
4.0	0.58720	0.591	3.6256	1.5436	0.4341	0.6478	0.7068
8.0	0.597163	0.602	7.534	1.5771	0.4556	0.6341	0.6709
64.0	0.62741	0.635	63.44	1.5799	0.4574	0.6329	0.6437

Figures 16 and 17 present values as a function of distance for $n \geq 1$ and $n \leq 1$; that is, the curves present $f(x)$ as a function of $\sqrt{x}$. The integral of the value $g(x)$ is presented in Figure 18 as a function of distance $\sqrt{x}$ for the extreme values of n considered. In Figure 19 the fraction survivors as a

[1] The "exact" values of β and α are found through solving the equation

$$g(1) = 1 - \frac{1}{e} = \int_0^\beta \exp(-y^n)\, dy .$$

This was accomplished numerically by evaluating the integral by Simpson's rule using 1,000 steps for differing values of β until the desired value was obtained. Since at n=1, β should be 1 exactly, the quoted value gives an indication of the error in this numerical procedure.

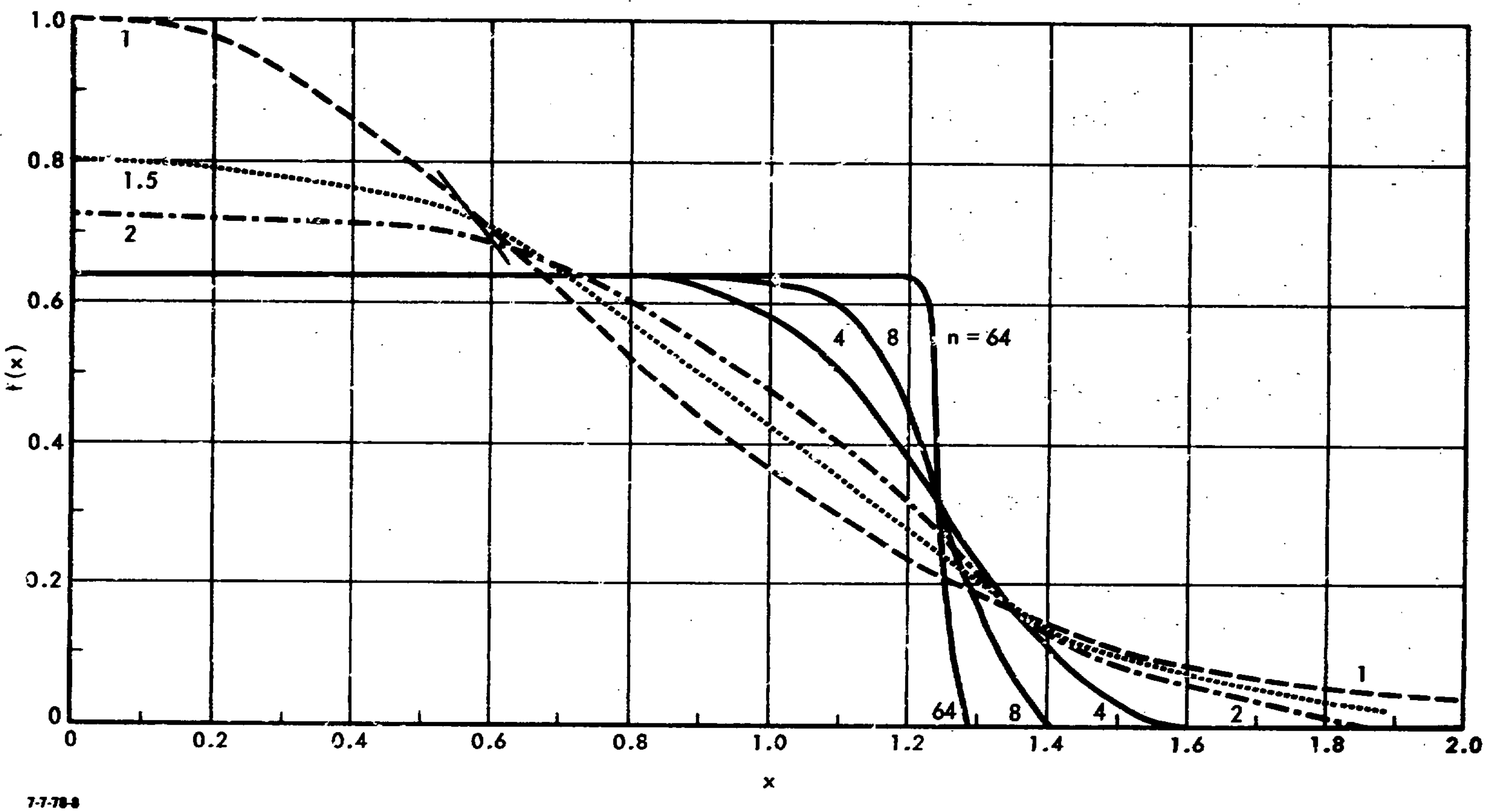

Figure 16. VALUE AS A FUNCTION OF DISTANCE FOR n GREATER THAN 1

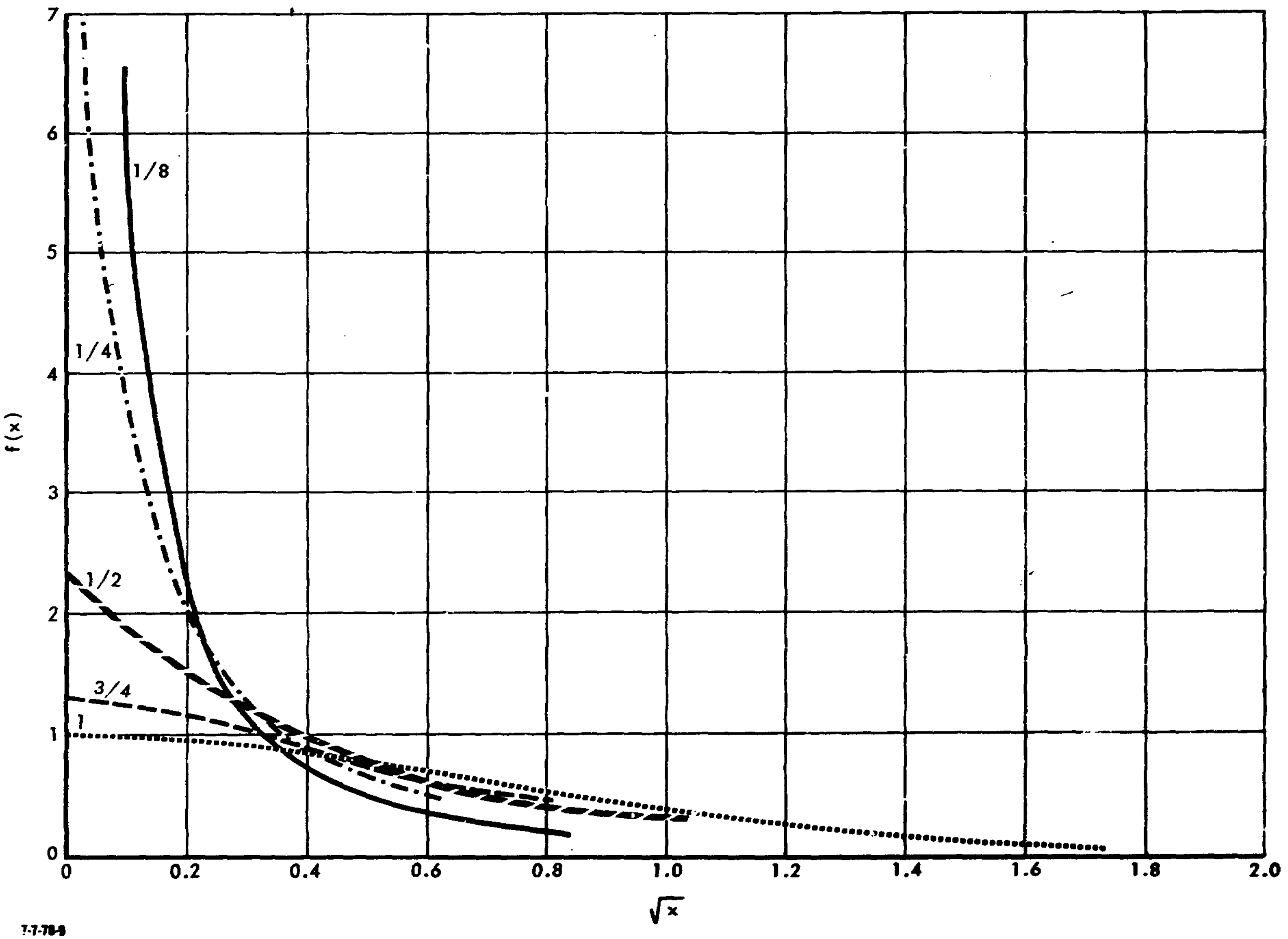

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Figure 17. VALUE AS A FUNCTION OF SQUARE ROOT OF DISTANCE FOR n LESS THAN 1

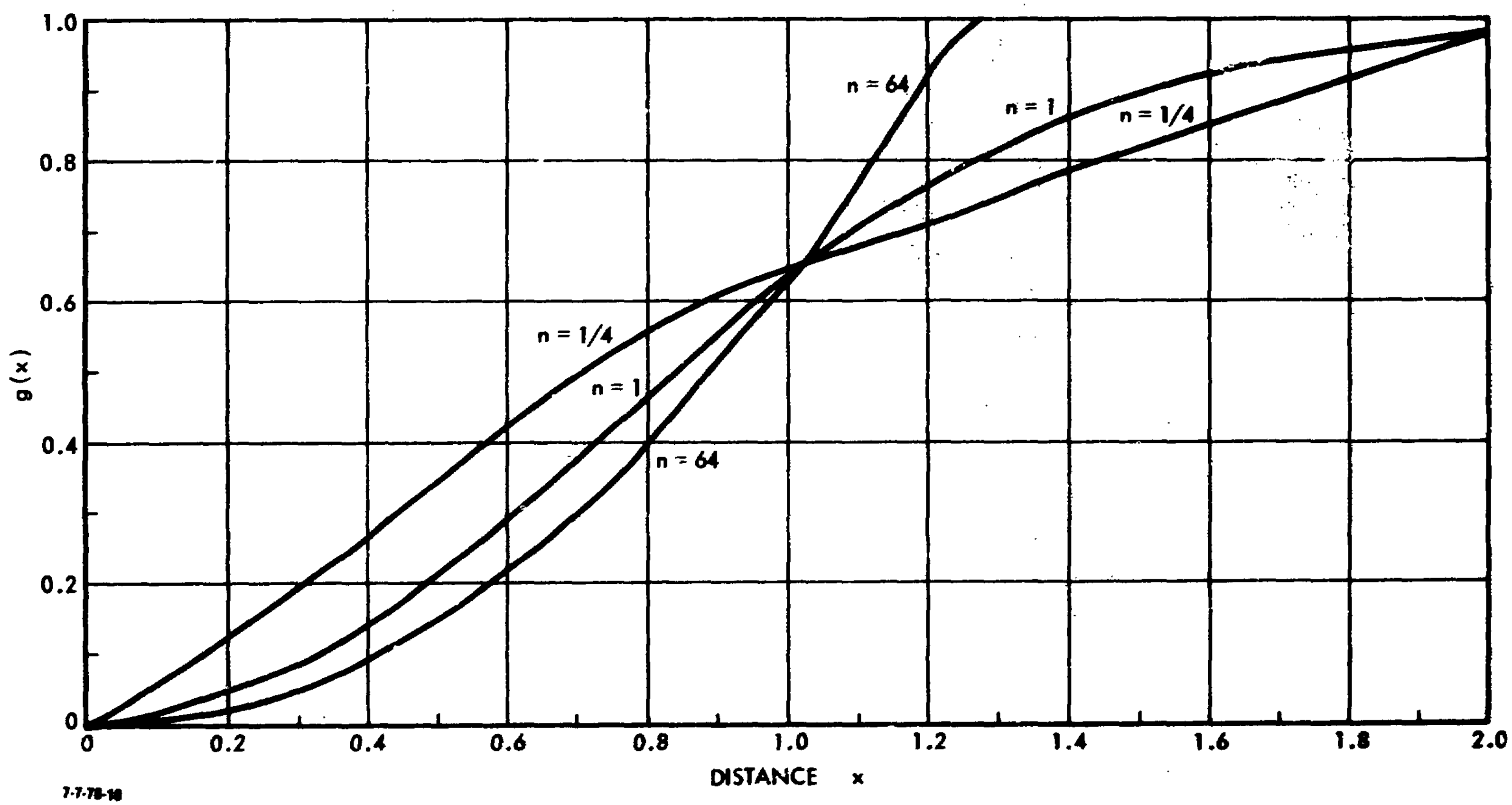

Figure 18. INTEGRATED VALUE AS A FUNCTION OF DISTANCE

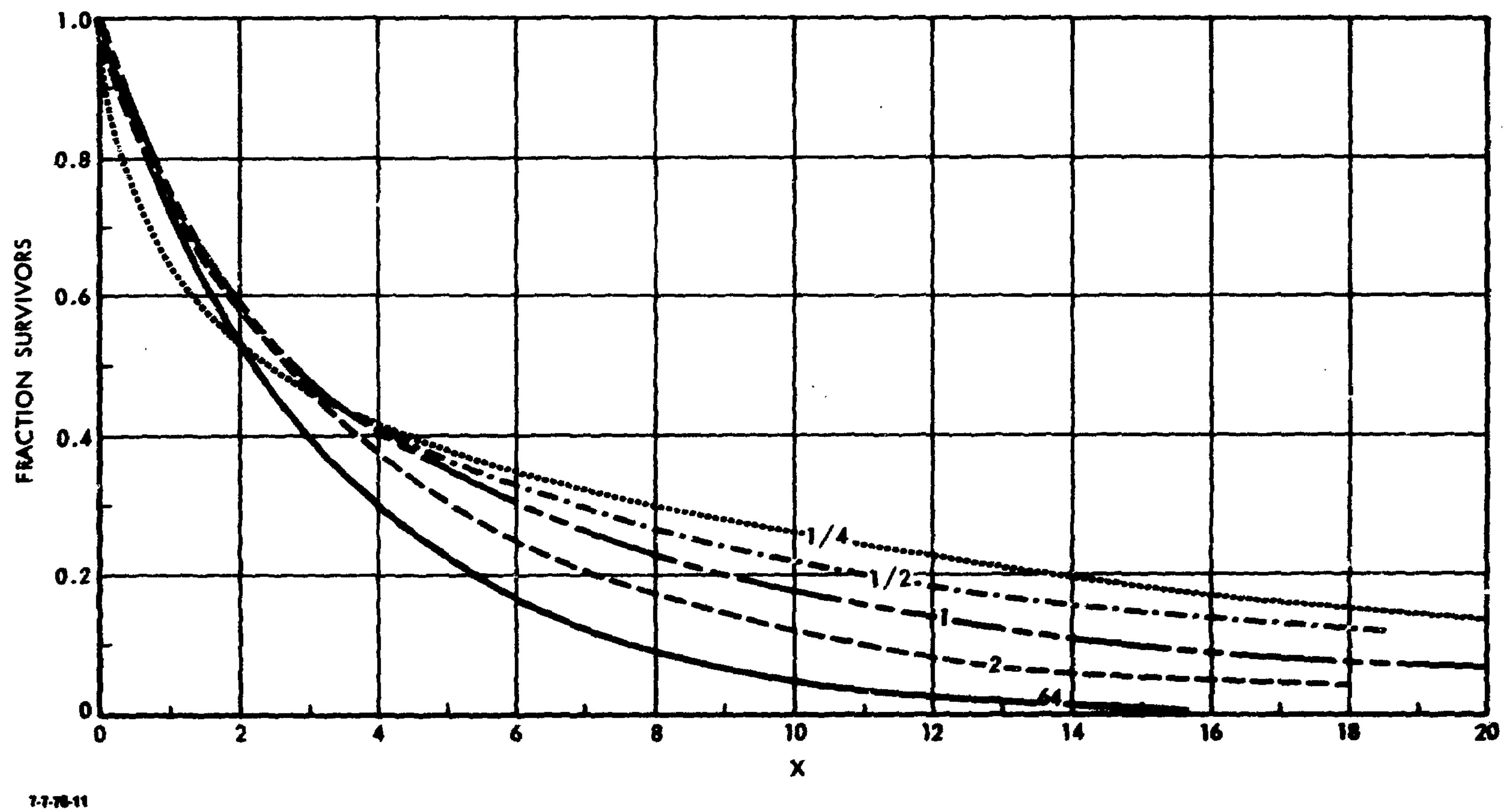

Figure 19. FRACTION SURVIVORS AS A FUNCTION OF WEAPON USAGE FOR VARIOUS VALUES OF n

function of normalized weapon usage is presented. For $n = 1$, the curve is the usual square root damage law. For $n = 64$, the value is almost uniform and then rapidly drops so the damage is close to that of a uniform disk optimally targeted; i.e., the fraction survivors is an exponential function of weapon usage. For $n = 1/2$, the value is an exponential function of distance. For values of n less than $1/4$, no curves are presented since the decay of value with distance is extremely rapid and numerical integration using Simpson's rule over intervals (the algorithm implemented here for numerical evaluations) becomes tedious.

L. FACTORS INFLUENCING THE PROBABILITY OF KILL AS A FUNCTION OF WEAPON DENSITY

In this section some rational for the use of a weapon density is presented. This is done first by studying the survival probabilities when a larger number of weapons arrive at random in a large area. Next the probability of survival from weapons delivered in regular patterns in an area are considered. This is done analytically for weapons where the probability of kill for the weapon is one inside a weapon radius and zero outside, and numerically for several typical types of assumed weapon kill probabilities as a function of distance.

Suppose weapons are delivered in an infinite plane so that any point in the plane is as likely to receive a weapon as any other. Then in any specified area the number of weapons delivered is a Poisson process with the distribution of number of weapons given by

$$P(n) = e^{-\omega A} \frac{(\omega A)^n}{n!} , \tag{125}$$

where A is the size of the area. Since the expected number of weapons in a unit area is ω, we will call ω the weapon density.

Consider any monitor point in the plane and let this monitor point be surrounded by a set of N non-overlapping but touching annuli, where the j^{th} annulus is centered a distance r_j from the monitor point and has a thickness dr_j.

The area of the j^{th} annulus is given by

$$A_j = \pi(2r_j dr_j + dr_j^2) .$$

The probability of n weapons arriving in this annulus is

$$P(n) = e^{-\omega A_j}\frac{(\omega A_j)^n}{n!}$$

The overall probability of kill of the monitor point from weapons in the j^{th} annulus is the sum of probabilities of kill for one weapon arriving, two weapons arriving, etc., or

$$P_j = P_k(r_j)\omega A_j e^{-\omega A_j} + \ldots + \left(1 - (1 - P_k(r_j))^n\right)\left(\omega A_j \frac{e^{-\omega A_j}}{n!}\right)^n .$$

The rings form a mutually exclusive set of areas covering the plane, except for a distant final outer section surrounding the rings. Then the probability of survival of the monitor point is given by the product of the probabilities of surviving the weapon arrival in each ring.

Now let N become indefinitely large so (1) for all j for $j = 1,2,\ldots N$, $\lim_{N\to\infty} A_j = 0$ and (2) $\lim_{N\to\infty} r_N = \infty$. It is clear that this can be done with a countable set of rings; for example, let $r_j = j/N^{3/4}$ $j = 1,2,\ldots N$. Then the area of each ring is

$$A_j = \pi\left(\frac{2j}{N^{3/2}} + \frac{1}{N^{3/2}}\right) < \pi\left(\frac{2}{N^{1/2}} + \frac{1}{N^{3/2}}\right)$$

and $\lim_{N\to\infty} A_j = 0$.

Then

$$S = \lim_{N\to\infty} \prod_{j=1}^{N} \left[1 - P_k(r_j)\omega A_j e^{-\omega A_j} + O(A_j{}^2)\right]$$

$$= \lim_{N\to\infty} \prod_{j=1}^{N} \left[1 - P_k(r_j)\omega 2\pi r_j dr_j + O(dr_j{}^2)\right] .$$

Now taking logarithms of both sides,

$$\ln S = \ln \lim_{N\to\infty} \prod_{j=1}^{N} \left[1 - P_k(r_j)\omega 2\pi r_j dr_j + O(dr_j{}^2)\right]$$

$$= \lim_{N\to\infty} \sum_{j=1}^{N} \ln(1 - P_k(r_j)\omega 2\pi r_j dr_j + O(dr_j{}^2)) .$$

Now

$$\ln (1-x) = -(x + \frac{x^2}{2} + \frac{x^3}{3} + \dots) \qquad |x| < 1 .$$

Thus,

$$\ln S = \lim_{N\to\infty} \sum_{j=1}^{N} -P_k(r_j)\omega 2\pi r_j dr_j + O(dr_j{}^2) .$$

Now, assuming the $P_k(r_j)$ are such that the sum converges, which is certainly true for all practical damage functions,

$$\ln S = \int_0^\infty -P_k(r_j)\omega 2\pi r_j dr_j$$

$$= -\omega \int_0^\infty P_k(A)\, dA . \tag{126}$$

Now $\int_0^\infty P_k(A)\, dA$ is often equated to πWR^2 where WR is called the weapon radius. The circular area is the area of a cookie cutter damage function which does the same damage. Thus, finally,

$$S = e^{-\omega\pi(WR)^2} \tag{127}$$

and

$$P = 1 - e^{-\omega\pi(WR)^2} .$$

This is the form of the damage function of Equation 2 which underlies the square root law formulation.

A suggestion by John Donelson[1] gives the same result in a much shorter fashion and in a way which makes the answer intuitively clear. The expected fraction of the plane covered by lethal effects is $\omega\pi(WR)^2$. A small change in this fraction, i.e., $\Delta\omega\pi(WR)^2$ gives a proportional fraction change in the survivors. So

$$-\frac{\Delta S}{S} = \Delta\omega\pi(WR)^2 , \tag{128}$$

or

$$-\Delta \ln S = \Delta\omega\pi(WR)^2 ,$$

so

$$S = e^{-\omega\pi(WR)^2} .$$

The random pattern of Poisson arrivals just discussed might be appropriate for an attack against certain types of valuable targets in an area where population happens to be calculated, and where the location of the population is not significantly influenced by the location of the other valuable targets. On the other hand, if an attacker were interested in

[1]Formerly with Institute for Defense Analyses, currently associated with Science Applications, Incorporated.

attacking an entire urban area, then he might place his weapons in a regular pattern. Any target in the urban area is somewhere within the pattern, but unless some reason is given to fix the location of the weapon pattern, the target may be anywhere within it.

Suppose weapons are placed at the intersections of a square grid with mesh length ℓ. Then the weapon density ω is simply $\omega = 1/\ell^2$. If the probability of kill is 1 inside some weapon radius WR, and 0 outside, then the expected kill is simply determined by the fraction of the plane covered by weapon circles. Call $\rho = WR/\ell/2$ the fraction coverage, and expected kill is readily seen to be

$$F = \begin{cases} \frac{\pi}{4}\rho^2 , & 0 \le \rho < 1 \\ \sqrt{\rho^2 - 1} + \frac{\rho^2}{2}\left(\frac{\pi}{2} - 2\cos^{-1}\left(\frac{1}{\rho}\right)\right) , & 1 \le \rho < \sqrt{2} \\ 1 , & \sqrt{2} < \rho \end{cases} \qquad (129)$$

The fatalities can be given as a function of the fraction coverage of the plane, C, to be consistent with the previous section. Then

$$C = \pi(WR)^2\omega = \frac{\pi}{4}\rho^2 . \qquad (130)$$

If the weapons are dropped in a more closely packed pattern of equilateral triangles of side length ℓ, then $\omega = 2/\sqrt{3}\ell^2$. Again, calling $\rho = WR/\ell/2$ for fraction fatalities,

$$F = \begin{cases} \frac{\pi}{4}\rho^2 , & 0 \le \rho < 1 \\ \sqrt{\rho^2 - 1} + \frac{\rho^2}{2}\left(\frac{\pi}{3} - 2\cos^{-1}\left(\frac{1}{\rho}\right)\right) , & 1 \le \rho < \frac{2}{\sqrt{3}} \\ 1 , & \rho \le \frac{2}{\sqrt{3}} \end{cases} .$$

The coverage is given by $C = \frac{\pi}{2\sqrt{3}}\rho^2$.

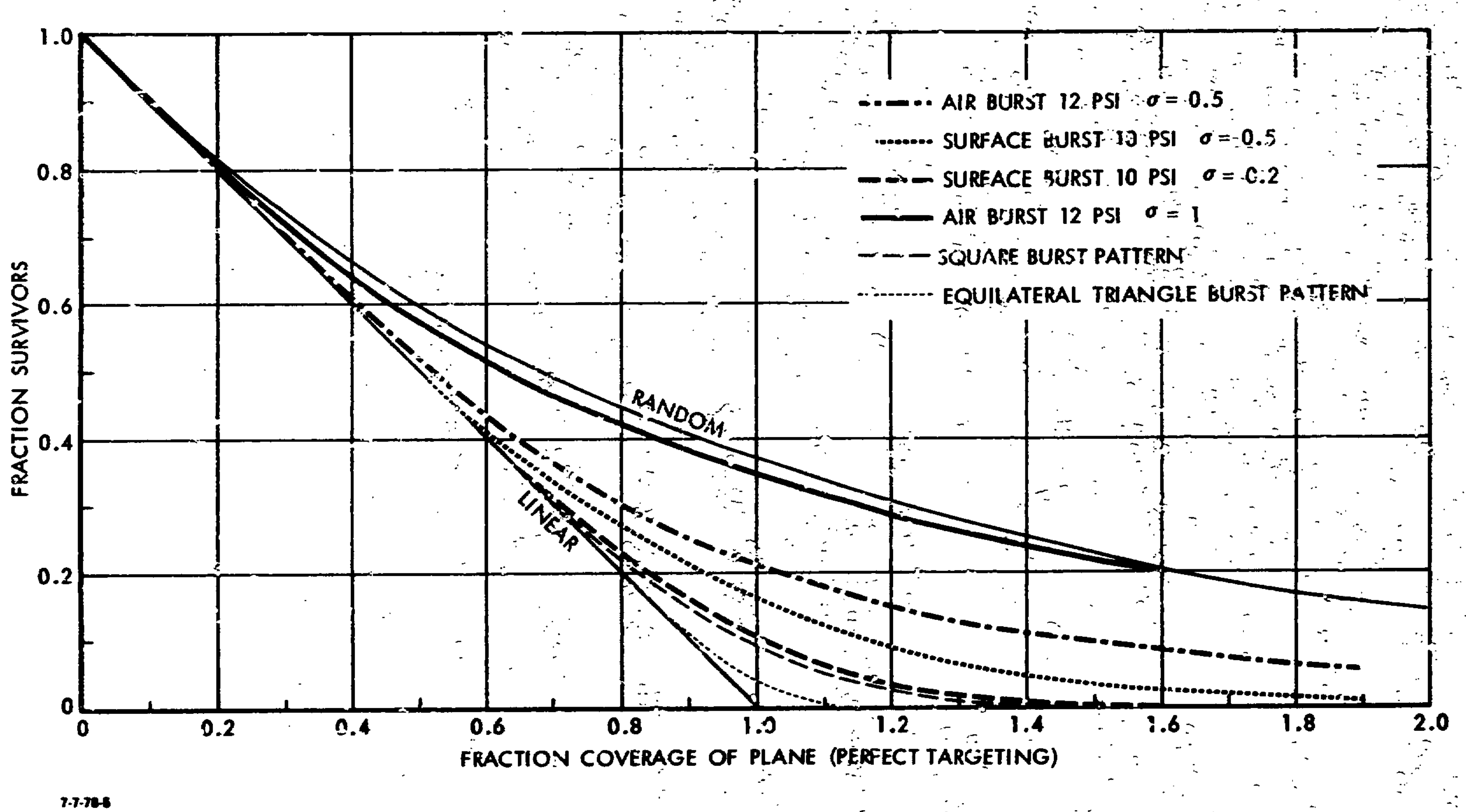

Figure 20. LINEAR PLOT OF SURVIVCRS AS A FUNCTION OF FRACTION COVERAGE FOR SEVERAL TYPES OF ATTACK

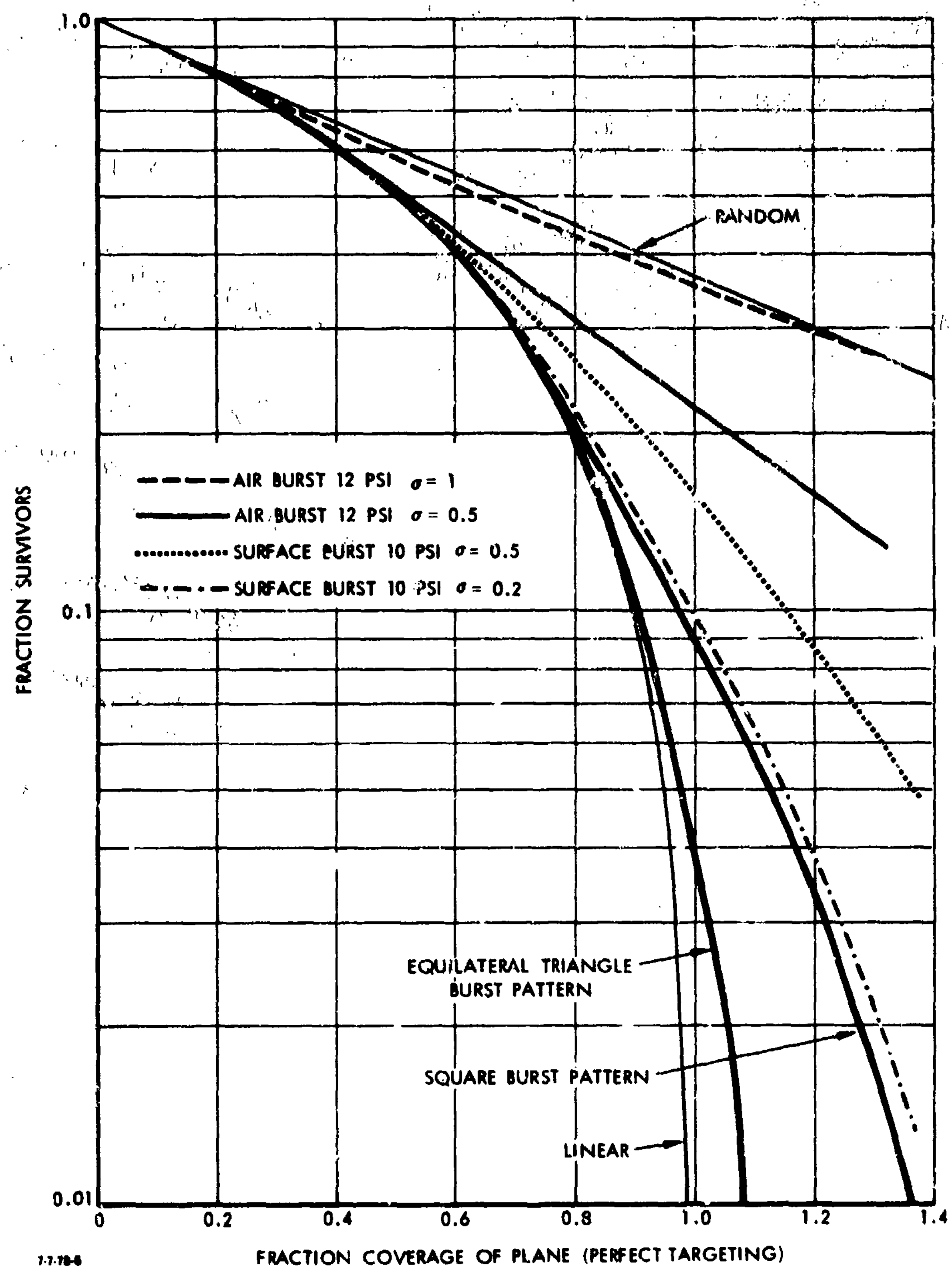

Figure 21. SEMI-LORARITHMIC PLOT OF SURVIVORS AS A FUNCTION OF FRACTION COVERAGE FOR SEVERAL ATTACK TYPES

The fraction survivors as a function of coverage is presented as a linear plot in Figure 20 and as a semi-logarithmic plot in Figure 21. Also presented are survivors for the random delivery considered earlier. The bottom straight line in Figure 20 represents the survivors if weapon effects could be perfectly placed, e.g., square lethal effects areas covering the plane for weapons placed in the center of each square. As can be seen, there is no overlap for the equilateral triangle pattern until 0.9 of the plane is covered by weapons effects. Even then there is less than four percent difference between this coverage and the no overlap variation.

If the fraction damage as a function of distance is a gradually decreasing function of the distance, then effects of overlap must be considered. Figures 20 and 21 show calculations for four cases of Figure 22 with weapons targeted in squares. In Figures 19 and 20 it can be seen that these cases range from the equivalent of almost perfect placement to almost random placement.

Some typical fraction damage as a function of distance examples are shown in Figure 22. They represent a range from very steep cutoffs in weapon lethality to quite gradual ones. The labels give an indication of the type of weapons target combinations considered. In obtaining these curves fraction kill as a function of distance is obtained by combining a function of fraction damage as a function of overpressure with a function of overpressure as a function of distance. The fraction of damage as a function of overpressure is assumed to be a cumulative log normal curve and the value σ in Figure 22 is the ratio of the standard deviation of the cumulative log normal curve to the mean lethal overpressure.

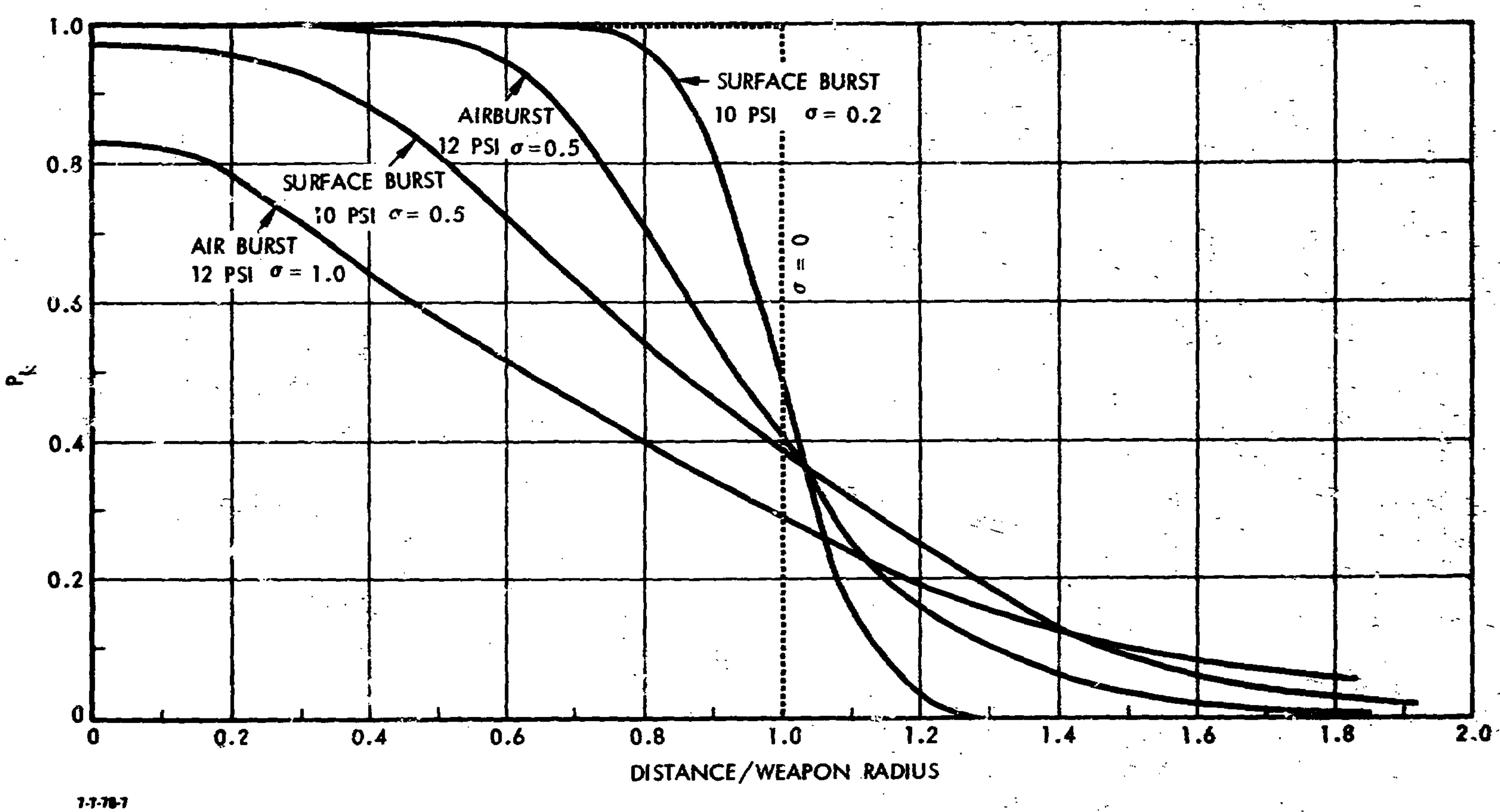

Figure 22. PROBABILITY OF KILL AS A FUNCTION OF NORMALIZED DISTANCE FOR SEVERAL KILL FUNCTIONS

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Analytical Representatives of Blast Damage for Several Types of Targets, (IDA Paper P-1373) by Leo A. Schmidt, Jr., Unclassified, Institute for Defense Analyses, October 1978, 67 pages, (Contract DCPA01-77-C-0215, Work Unit 4114G)

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